1957

Application of correlation detection to distance measuring equipment.

Sheridan, William R.

Monterey, California: Naval Postgraduate School, 1957.

APPLICATION OF CORRELATION DETECTION TO DISTANCE MEASURING EQUIPMENT

WILLIAM R. SHERIDAN

APPLICATION OF CORRELATION DETECTION TO

DISTANCE MEASURING EQUIPMENT

* * * * * * * *

William R. Sheridan

Released for general distribution by letter from Cubic Corp. 6/27/57

APPLICATION OF CORRELATION DETECTION TO DISTANCE MEASURING EQUIPMENT

by

William R. Sheridan

Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
ENGINEERING ELECTRONICS

United States Naval Postgraduate School
Monterey, California

1957

APPLICATION OF CORRELATION DETECTION TO DISTANCE MEASURING EQUIPMENT

by

William R. Sheridan

This work is accepted as fulfilling

the thesis requirements for the degree of

MASTER OF SCIENCE

IN

ENGINEERING ELECTRONICS

from the

United States Naval Postgraduate School

ABSTRACT

Ballistic Missiles have been developed or are in the offing which are capable of long ranges and extremely high velocities. In order for these missiles to perform their purpose most effectively, accurate range and velocity instrumentation equipment is needed. A system has been developed to accurately position ballistic missiles by Cubic Corporation of San Diego, California. Improvements on the present system are desired, particularly in the field of minimum detectable signal level and recovery of velocity data. A system is described herein which will realize an improvement over the present system. The proposed system is compared with the present system and appears very favorable theoretically. Production-wise, the proposed system is feasible.

The writer wishes to express his appreciation for the assistance and encouragement given him by Professors C. F. Klamm, Jr., and R. L. Miller of the U. S. Naval Postgraduate School in this investigation.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

1. Introduction

Cubic Corporation of San Diego, California has in operation electronic equipment which is capable of accurately ranging upon high velocity and highly maneuverable ballistic missiles. An interrogator and transponder arrangement is employed; the transponder being located in the missile and the interrogator residing at a ground station. Slant range over the interval of less than a mile to 200 miles is measured with a maximum rms error of 2.5 ft. Range rates of over 20,000 ft. per second can be handled over the range interval with the aforementioned accuracy.

The basic principle of operation is couched in a precision phase comparison between the interrogator transmitted signal and the signal received at the interrogator from the transponder. The equipment operates in the UHF region. A UHF carrier is frequency modulated by a comparatively low frequency sine wave at the interrogator and transmitted omnidirectionally. When the signal is intercepted by the transponder, it is effectively linearly translated in the frequency domain and reradiated omnidirectionally. This signal is then received by the interrogator, demodulated and a phase comparison is made between the demodulated signal and the original modulating sine wave in a servo phase meter. Transponder range is a direct function of the phase difference between the demodulated signal and the original modulating sine wave. The above process may be described analytically as follows:

The generated signal at the interrogator may be expressed as

$$\cos(\omega_0 t + a(t))$$

where ω_0 = carrier frequency expressed in radians/sec.

$a(t)$ = modulating intelligence

The signal received at the transponder is given by

$$\cos(\omega_0(t-T) + a(t-T))$$

where T = time delay in reception of interrogator signal by the transponder

The signal transmitted by the transponder is given by

$$\cos((\omega_0 - \omega_i)(t-T) + a(t-T))$$

where ω_i corresponds to the frequency shift carried out at the transponder

The signal received by the interrogator is then

$$\cos((\omega_0 - \omega_i)(t-T-T) + a(t-T-T))$$

After demodulation, the receiver output is

$$a(t-T-T)$$

Let $a(t) = \sin \omega_m t$

$$a(t-2T) = \sin(\omega_m t - 2\omega_m T)$$

The output of the phase meter is proportional to the phase difference between $a(t)$ and $a(t-T)$, that is $\omega_m T$. This is a direct function of range. The phase meter output is calibrated to give true range in feet.

It is to be noted that there may be cyclic ambiguities which must be resolved. Ambiguity resolution is obtained by modulating the carrier with several sine waves. The frequencies of the respective sine waves are: 491.76 kc, 61.470 kc, 7.68375 kc, 1.92094 kc, and .192 kc. Hereinafter, they will be referred to as the modulation subtones.

As has been stated, the type modulation being used is frequency

modulation. The principle reason for use of this type of modulation is the relatively large post detection signal to noise ratio improvement that can be realized with wide band angle modulation. There is nothing unique in the desired information bearing content of the FM signal. An AM signal could just as well have been used in so far as the desired information bearing content is concerned. The decision to use FM was primarily based on the quality of information recovery.

The equipment was primarily designed to be used in an over-all system that accurately positions ballistic missiles. To position an object in space, three parameters must be known in any one of the following four combinations: 1) three ranges, 2) three direction cosines, 3) two ranges and one direction cosine, 4) two direction cosines and one range. Range information is determined, in a manner described heretofore, by a distance measuring equipment, hereinafter referred to as a DME. Angle information is determined by an angle measuring equipment, commonly called an AME.

The basic principle of operation of an AME is again a phase comparison; however, it is a phase comparison between a received carrier signal arriving at two different points. Ambiguity resolution is obtained by spacial distribution of receiving antennae.

This paper will be concerned with a fundamental problem inherent in the DME. Sufficient operational characteristics have been presented in order to intelligently formulate the problem and propose a solution. Specific details concerning the AME or the DME may be found in references (1) and (2). Suffice it to say that Cubic Corporation has in operation accurate ranging equipment capable of giving instantaneous range vs. time data. A further noteworthy fact is that this equipment is relatively simple with regard to equipment complexity; and is relatively inexpensive.

2. Formulation of the Problem

The problem at hand is twofold. First, there is information concerning instantaneous radial missile velocity inherently present in the received signal at the ground station. Means should be developed to extract velocity information as well as obtaining range information. The interrogator transmitted carrier frequency is known precisely. The interrogator received carrier frequency differs from the transmitted frequency by the pertubation carried out at the transponder plus doppler shift due to transponder velocity. If transponder carrier frequency pertubation is known precisely, and if the interrogator received carrier frequency can be detected in some manner, it can be compared with the interrogator transmitted carrier frequency. The difference frequency will be due to doppler shift plus some predetermined pertubation which can be calibrated out of the system. With these conditions satisfied, a continuous measurement of interrogator transmitted and received carrier frequencies is equivalent to a continuous measurement of doppler shift. Hence, instantaneous radial velocity vs. time data is available. By suitable calculations upon three radial velocities obtained from three DME ground stations a resultant velocity vector may be obtained.

It may be argued that velocity information can be obtained from the time average of the range data extracted from the equipment. This, to be sure, is a valid argument; however, the velocity information calculated by this method would be an average velocity over a given time interval and thus would not be instantaneous. Furthermore, this method would be subject to a fairly large error under conditions of missile acceleration.

The first problem imposes two requirements upon a proposed solution:

1. The linear transformation in the frequency domain, of the transponder received signal, must be precisely predetermined.
2. The carrier frequency, in addition to the modulation subtones, must be abstracted from the signal received from the transponder by the interrogator.

The imposition of the second requirement appears to imply that a carrier frequency must be initially transmitted by the interrogator. This is not true in all cases. It will be shown that in suppressed carrier amplitude modulation, the would be carrier can be obtained by suitable receiver operations on the sidebands.

The second problem, and probably the more important one, consists of incorporating into the DME a more efficient means of signal detection. A solution to the problem will have far reaching implications upon the DME system as a whole. For this reason, a further discussion of this problem is appropriate.

Signal detection, at the interrogator receiver, is presently being accomplished by means of a conventional frequency discriminator. It is well known that such detection is subject to a threshold effect as predetection noise power level approaches the same order of magnitude as the predetection signal power level. In the DME threshold occurs at a signal to noise ratio of approximately 10 db. As the signal to noise ratio decreases below this value, the useful output of the detector deteriorates rapidly. This deterioration cannot be eliminated by post detection filtering. The degradation of signal to noise ratio by means of conventional detection is an irreversible process.

In most communication systems, post detection bandwidth is more narrow than predetection bandwidth. This statement is particularly

true in the case of the DME; for, the detected signals are five discrete sinusoids sufficiently separated in the frequency domain to be individually filtered. The noise bandwidth, however, is determined by the receiver predetection bandwidth. In the DME, the predetection bandwidth is approximately 2 megacycles. Individual post detection bandwidths are considerably less than this figure and could be made even more narrow if crystal and mechanical filters were employed.

Were it possible to reduce the value of signal to noise ratio whereupon threshold occurs or possibly eliminate the threshold effect entirely, the relative signal level required may be reduced. This reduction may be projected back to the transponder transmitter. This implies that greater ranging capabilities may be realized while utilizing the same transponder power output; or, the same ranging capabilities may be obtained with a reduction of transponder power output. These are the implications of a more efficient means of signal detection.

In summation, the problems to be investigated are:

1. Incorporate in the DME system means of obtaining instantaneous velocity vs. time data.
2. Incorporate in the DME system a more efficient means of signal detection.

The solution to the above problems must be in harmony with the cost and complexity of the present system.

3. Correlation Detection - A Quasi Heuristic Solution

It could be stated that DME ranging performance is essentially limited by noise that enters the system. The detection of signals in noise leads one to the consideration of correlation techniques. Much literature has been written and published during the last ten years concerning the detection of signals in the presence of noise by means of correlation techniques. Specific references to these articles would indeed require a voluminous bibliography.

Many schemes employ autocorrelation or crosscorrelation directly, that is, computing $\phi_{11}(\tau)$ or $\phi_{12}(\tau)$. From the definitions of these functions, the basic processes executed in both cases are those of multiplication and time averaging. When applied to the present problem, multiplication of itself appears to offer no difficulty; however, time averaging may raise a serious objection. Time averaging could produce sizeable errors in system data, particularly when tracking high speed missiles.

It may be shown quite simply that forming the autocorrelation function of the received signal is not in general a good means of detecting the signal. Let the received signal be

$$s(t) + n(t)$$

where $n(t)$ is additive white gaussian noise

Forming the autocorrelation function gives

$$\phi_{11}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} [s(t) + n(t)][s(t+\tau) + n(t+\tau)]\,dt$$

$$= \overline{s(t)\,s(t+\tau)} + \overline{n(t)\,s(t+\tau)} + \overline{s(t)\,n(t+\tau)} + \overline{n(t)\,n(t+\tau)}$$

where the bar signifies time averaging.

$$\phi_{11}(\tau) = \phi_{ss}(\tau) + \phi_{ns}(\tau) + \phi_{sn}(\tau) + \phi_{nn}(\tau)$$

Since the noise and signal are uncorrelated

$$\phi_{ns}(\tau) = \phi_{sn}(\tau) = 0$$

The correlator output is then $\phi_{ss}(\tau) + \phi_{nn}(\tau)$

$\phi_{nn}(\tau)$ is an undesireable noise term. An infinite averaging time has been assumed. For a non-infinite averaging time.

$$\phi_{ns}(\tau) \neq \phi_{sn}(\tau) \neq 0$$

Thus, in a practical system, there will be three noise terms.

Consider forming the crosscorrelation function between $s(t) + n(t)$ and $s(t)$.

$$\phi_{12}(\tau) = \overline{[s(t) + n(t)]\, s(t+\tau)}$$

$$= \phi_{ss}(\tau) + \phi_{ns}(\tau)$$

$$= \phi_{ss}(\tau)$$

Thus computing the crosscorrelation function between the input signal plus noise, with the signal, produces the autocorrelation function of the signal. This, however, requires the signal to be known exactly, and would not appear to be suitable for target acquisition. Furthermore, an infinite averaging time is required to reduce $\phi_{ns}(\tau)$ to zero.

The mathematical operations performed in correlation techniques are multiplication and time averaging. In the frequency domain, this consists of a spectrum shift and filtering. The fundamental unit of a correlation detector is a multiplier in which the incoming signal is multiplied by some multiplying quantity continuously delayed in time. This, then, will be the fundamental unit to be proposed in the DME interrogator receiver. The multiplying quantity will be the carrier frequency of the received signal, not continuously delayed in time, but with a fixed time delay with respect to the received carrier frequency. This has been chosen since not only are the elements of

correlation detection present, but also, the multiplying quantity is available for comparison with the interrogator transmitted carrier frequency in order to obtain velocity data. The filtered output of the multiplier will be, statistically speaking, the coefficient of linear correlation between the received signal and the locally supplied signal. It may be noted that this type of detector is not new in its conception. Papers as early as 1922 (9) have been published concerning this detector. Through the years, such names as product demodulator, synchronous detector, correlation detector and coherent detector have been used in describing the same detector. This detector is shown in Fig. 1 and henceforth will be referred to as a synchronous detector.

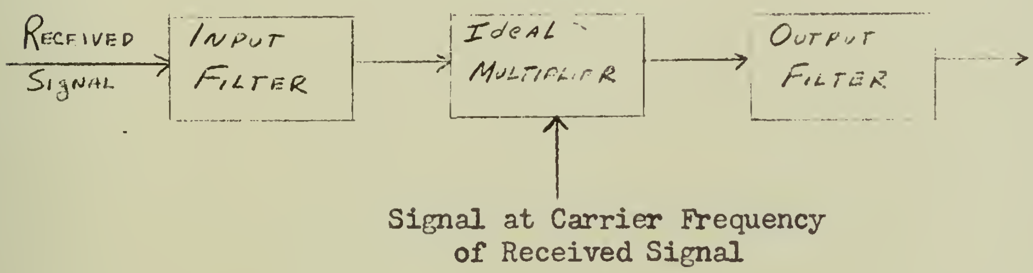


Fig. 1 - Block Diagram of the Synchronous Detector

It will be shown in subsequent analyses that incorporation of the synchronous detector in the DME will greatly enhance the overall operational characteristics of the system. This improvement in equipment performance lies in the fact that better utilization can be made of a given signal density at the receiver input. From this standpoint the DME could be considered more efficient. It is to be expected that equipment complexity and cost will increase. This follows from what may be ennunciated as a general statement concerning physical systems: the higher the performance desired of a system, the higher the equipment complexity and cost. Note, the term "general statement", was used, not "universal statement"; hence

exceptions may be admitted. It is felt that any increased equipment complexity and cost is relatively conservative and therefore in harmony with the general complexity and cost of the present system.

The synchronous detector appears to solve the twofold problem subject to the cost constraint. The extent to which the problem is solved is discussed in subsequent analyses.

The approach to the problem has been heuristic in nature. That is, an answer was assumed and found to be satisfactory. However, one might inquire, is this the best solution? In particular, is the best or optimum detection scheme being employed? Answers to these questions require a more philosophical approach to the theory of signal detectability. This will be investigated in a later section.

4. Fundamental Properties of the Synchronous Detector

Consideration of some fundamental properties of the synchronous detector will give an insight into proper application of the detector. Appendix I gives an analysis of the synchronous detector from a $\frac{S}{N}$ ratio point of view.

From Fourier Transform Theory, the spectrum of the product of two time functions is the convolution of the individual spectra. When applied to the synchronous detector the received signal is convoluted with a pair of symmetrical delta functions. The spectrum of the multiplier output (considering the low frequency portion only) consists of the input spectrum shifted to the origin in the frequency domain. The nonlinear operation of multiplication in the time domain is a linear transformation in the frequency domain. If the spectrum of the received signal is the spectrum of the intelligence "delayed in frequency," then by the uniqueness of Fourier Transform Pairs, the low pass filter output is identically the intelligence.

It can be seen from Appendix I that the output $\frac{S}{N}$ ratio is not a function of predetection bandwidth. Ideally, with proper phase synchronization, the output $\frac{S}{N}$ ratio is dependent only upon input noise power spectral density and post detection filtering. Thus the synchronous detector is characterized by having no detection threshold, as the term is generally understood. Further, when the input noise power is considered as that which is determined by the post detection filter there is $\frac{S}{N}$ ratio linearity through the device.

Since the output $\frac{S}{N}$ ratio is determined entirely by post detection filtering, there appears to be no reason for considering any bandwidth greater than the post detection bandwidth in so far as $\frac{S}{N}$ ratios are concerned. The receiver noise bandwidth is the effective noise bandwidth

of the individual post detection filters in lieu of the IF amplifier noise bandwidth. The signal power at the receiver input is not that required to overcome the noise power determined by the IF bandwidth but only that which is required to overcome the noise power determined by the post detection filter bandwidth. The extent that the latter can be made smaller than the former is a measure of the signal power reduction which can be realized at the receiver input.

The fact that the $\frac{S}{N}$ ratio at the detector output is a direct function of the post detection bandwidth has been recognized by Fano (3) and Woodward (10). $\frac{S}{N}$ ratio linearity through the device can be considered a correlary of this finding.

The synchronous detector has been shown to be an efficient demodulator. However, this is subject to the assumption that the spectrum of the received signal is the spectrum of the intelligence "delayed in frequency". This assumption has an important implication with regard to the type modulation best suited for reception. Speaking in terms of the frequency domain, synchronous detection amounts to a linear operation on the received spectrum. It seems quite natural that in order to recover the intelligence spectrum, the received signal spectrum should be a linear function of the intelligence spectrum. For amplitude modulation this is certainly the case; for, in amplitude modulation, the spectrum of the signal is the intelligence spectrum "delayed in frequency" by an amount equal to the carrier frequency. However, for frequency modulation in general, the spectrum of the signal does not bear any linear relationship to the intelligence spectrum. Instead, the spectrum is a sinusoidal function of the intelligence spectrum "delayed in frequency". Although for narrow band frequency modulation with a

modulation index, δ sufficiently small such that $J_1(\delta) \approx \frac{\delta}{2}$ and $J_n(\delta) \approx 0$ where $n = 2, 3, 4 \ldots$, the spectrum of the signal is the spectrum of the intelligence "delayed in frequency". A more detailed analysis of synchronous detection of frequency modulated signals and amplitude modulated signals will be given in the following sections.

Since the synchronous detector possesses the unique property of $\frac{S}{N}$ ratio linearity, while at the same time acting as a demodulator for certain signals, it is of interest to know what $\frac{S}{N}$ ratio is required at the receiver output in order to execute a reliable phase comparison measurement between the reference modulation subtone and the detected modulation subtone. Experimental data was obtained from a servo phase meter unit in readiness for shipment to the contractor. The data is presented in Fig. 2. The data was taken for the 491 kc channel only, since the channel is, so to speak, the vernier of range measurement. All channels are essentially identical except for the frequency at which the phase comparison was made. From the curve, it is felt that a $\frac{S}{N}$ ratio of approximately 10 db is required at each channel input to the servo phase meter.

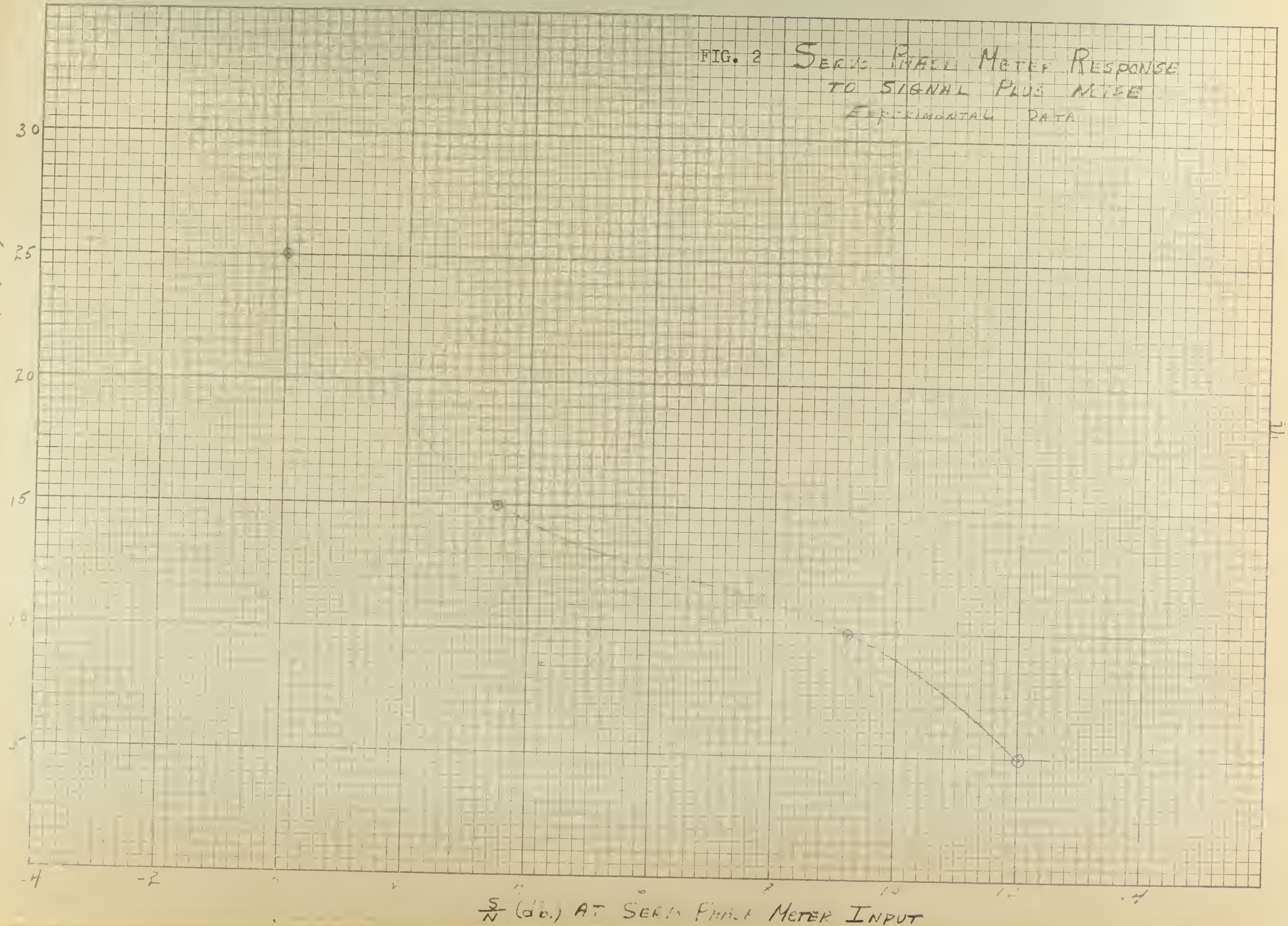


FIG. 2 SERVO PHASE METER RESPONSE TO SIGNAL PLUS NOISE
EXPERIMENTAL DATA

5. Synchronous Detection of FM Signals and its Relation to the Present Problem

Since Cubic Corporation's equipments utilize frequency modulation it is of prime interest to further investigate the operation of a synchronous detector when the input is an FM signal. Let the detector input signal be $\sqrt{2}\ A\sin(\omega_c t + a(t))$ and let the reinserted carrier be $\sqrt{2}\ C\cos(\omega_c t + \phi)$, where ϕ is the phase difference from quadrature between the reinserted carrier and the received carrier. The low frequency output of the multiplier is

$$AC\sin(a(t) + \phi) \qquad (1)$$

a(t) represents the intelligence which is desired at the output of the receiver, however, the actual receiver output is a sinusoidal function of the intelligence. It may be asked, is there a physically relizeable circuit which may operate upon $\sin(a(t) + \phi)$ to produce $a(t) + \phi$? One is very tempted to use the Paley-Wiener criterion. To be sure, this criterion will show that $a(t) + \phi$ cannot be exactly extracted from $\sin(a(t) + \phi)$. It is not very reasonable to expect that a linear circuit will operate on an input in a nonlinear fashion. A nonlinear operation must be performed on sin x in order to recover x, but by the mere fact of invoking the Paley-Wiener criterion a linear operation is demanded. Discussions with several engineers at Cubic Corporation familiar with the problem have failed to find a solution other than demand that the modulation index be sufficiently small such that sin a(t) can be approximated by a(t). A suggestion has been made by Professor C. F. Klamm, Jr. that it may be possible to use somewhat higher modulation indicies and recover the intelligence with tolerable distortion for the particular application. Or in other words, perform an operation on sin a(t) and

obtain a(t) plus some other terms wherein these other terms will have negligible effect on the phase parameter of a(t) which will eventually be required in measurement. This suggestion is well founded and deserves some consideration.

This author must state that a complete solution has not been found. Some thought has been given to the idea and the essentials will be presented here. Conceivably these could be used as a starting place for future investigations.

First consider the spectrum of a carrier frequency modulated by five discrete sinusoids. It can be shown that the frequency spectrum can contain the following frequencies:

carrier frequency, $\pm mf_1, \pm nf_2, \pm of_3, \pm pf_4, \pm qf_5$

where m,n,o,p, and q can assume all integral values independent of each other. The actual presence of a particular sideband is determined by the individual modulation indicies of all five of the discrete sinusoids. In FM the modulating voltages do not add energy to the resultant signal but distribute the energy present in the unmodulated carrier to the various sidebands.

It is very possible that the various modulation indicies could be adjusted such that the several modulation subtones may be individually filtered with tolerable adjacent sideband interference. An analytical approach to this, though simple enough in theory, would become quite tedious. All combinations of all plausible individual modulation indicies would have to be considered. The quickest method of solution would most probably be an experimental test.

Another approach would be to convert the FM signal to AM signal immediately prior to synchronous detection. The question immediately

arises, why not use AM in transmission? If FM is demanded by reasons unknown to the author, these approaches should receive attention.

A method has been suggested by the author to alleviate the constraint upon modulation index and at the same time use and enjoy the benefits of synchronous detection. Consider a(t) in equation (1) to be $\delta \sin \omega_m t$. Expansion of (1) gives

$$AC \sin \phi J_0(\delta) + 2AC \cos \phi \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} J_n(\delta) \sin n\omega_m t + 2AC \sin \phi \sum_{\substack{n=2 \\ n \text{ even}}}^{\infty} J_n(\delta) \cos n\omega_m t \quad (2)$$

For the synchronous condition, $\sin \phi = 0$ and the third term is eliminated. If the post detection filter is sufficiently narrow to discriminate against the third harmonic of f_m then the modulation index, δ should not be small but should be adjusted to the value wherein $J_1(\delta)$ is a maximum, for maximum receiver output. The value of δ to produce this is approximately 1.8. This method has been verified experimentally. The receiver output may be sent to the servo phase meter for a phase comparison measurement. A $\frac{S}{N}$ ratio analysis may be conducted in a similar manner to that performed for the amplitude modulation signal given in Appendix I. The end result will be essentially the same but with $J_1(\delta)$ replacing m, the modulation index for amplitude modulation. The question of system accuracy and resolution of cyclic ambiguities still remains a problem.

From what has been presented in this section, it is felt that synchronous detection of FM, as applied to the present DME, requires that the signal have a small modulation index. Henceforth, when speaking of FM, it will be understood that narrow band FM (small modulation index) is being referred to unless otherwise noted.

6. Comparison between FM and AM in the DME

Cubic Corporation has proposed a "coherent carrier" scheme for use in the DME. This proposal utilizes synchronous detection of FM with a modulation index $\ll 1$, such that sin a(t) can be approximated by a(t). It is of interest to compare synchronous detection of FM and AM in its relation to DME use. A suitable standard of comparison is of course needed. It is recognized that, in general, comparisons between physical systems do not always convey the complete picture and can be misleading. This is due for the most part in the artificiality of the standard of comparison or the inadequacy of standards. Notwithstanding this, a comparison will be attempted with the point of view that some information is better than no information.

As a starting place it may be asked, is there any effect on system ranging capabilities when AM is used in lieu of FM or vice versa? If so, power output may be increased for one method to obtain equal ranging capabilities. What then are the implications of increasing output power. At the interrogator transmitter it will in general mean larger equipment. In the range of presently employed power outputs this would not be too bothersome. At the transponder the output power must also be increased. In ballistic missiles a minimum amount of weight and space is allocated to the transponder. Any reduction of weight and space required for electronic equipment is greatly welcomed and sometimes demanded by the aeronautical engineer. Further, dissipation of heat generated by the transponder is an extremely critical problem. All of these factors are in general a function of output power. The greater the power, the greater these problems become. It is realized that there is not a direct relation between them, but there is some

relation. That is, doubling the power output at the milliwatt level would not be as troublesome as doubling the power output at the watt level. With the preceeding in mind, it appears that a logical standard of comparison may be total signal power. The interrogator-transmitter to transponder-receiver link will not be considered since this link is so to speak the "invulnerable" portion of the system. The transponder-transmitter to interrogator-receiver link will be spoken of from a total signal power point of view since we are really trying to make maximum use of the signal density arriving at the interrogator receiver. In the spirit of what has been presented, the following three items will be discussed:

A. For a given receiver gain and a given receiver output voltage requirement, what is the ratio of FM signal power to AM signal power required at the receiver input.

B. For a given transponder power output, what is the ratio of maximum range attainable when using FM or AM, assuming the receiver gain is the same for both types of modulation.

C. For a synchronous detector, what is the ratio of at the receiver output when the input is FM or AM, both of equal power.

It will be seen that these three items consist in viewing the same entity but from different points of interest. The fact that the results will appear the same does not necessarily make the analyses redundant; rather, it will serve to understand more fully any advantages of one system over the other.

A. Signal Power Requirements at the Receiver Input

The FM signal $e_{fm} = A \sin(\omega_c t + \delta \sin \omega_m t)$

$$e_{fm} = J_0(\delta) A \sin \omega_c t + J_1(\delta) A \sin(\omega_c + \omega_m)t - J_1(\delta) A \sin(\omega_c - \omega_m)t \quad (1)$$

Since $J_0(\delta) \approx 1$ and $J_1(\delta) \approx \frac{\delta}{2}$ for δ sufficiently small

$$e_{fm} = A \sin \omega_c t + \frac{\delta A}{2} \sin(\omega_c + \omega_m)t - \frac{\delta A}{2} \sin(\omega_c - \omega_m)t \qquad (2)$$

The AM signal $e_{am} = A' \sin \omega_c t + \frac{A'm}{2} \cos(\omega_c + \omega_m)t + \frac{A'm}{2} \cos(\omega_c - \omega_m)t$ (3)

Under the assumptions that the receiver has a fixed gain and a given receiver output voltage is required, the following must be true at all times

$$\delta A = m A' \qquad (4)$$

The total power in the FM signal $= \frac{A^2}{2}$

The total power in the AM signal $= \left(\frac{A'}{\sqrt{2}}\right)^2 + \left(\frac{mA'}{2\sqrt{2}}\right)^2 + \left(\frac{mA'}{2\sqrt{2}}\right)^2$

The ratio of FM to AM signal power is

$$\frac{A^2}{A'^2 + \frac{m^2 A'^2}{2}} \qquad (5)$$

Invoking equation (4) the signal power ratio is

$$\frac{P_{fm}}{P_{am}} = \frac{m^2}{\left(1 + \frac{m^2}{2}\right)\delta^2} \qquad (6)$$

The above ratio may be stated thusly: The amount of FM signal power, expressed in db, required in excess of the amount of AM signal power at the receiver input in order to obtain the same amount of data signal power at the receiver output is given by

$$10 \log \frac{m^2}{\left(1 + \frac{m^2}{2}\right)\delta^2} \qquad \text{where } \delta < 1, \quad 0 \le m \le 1 \qquad (7)$$

A plot of this function is shown in Fig. 3.

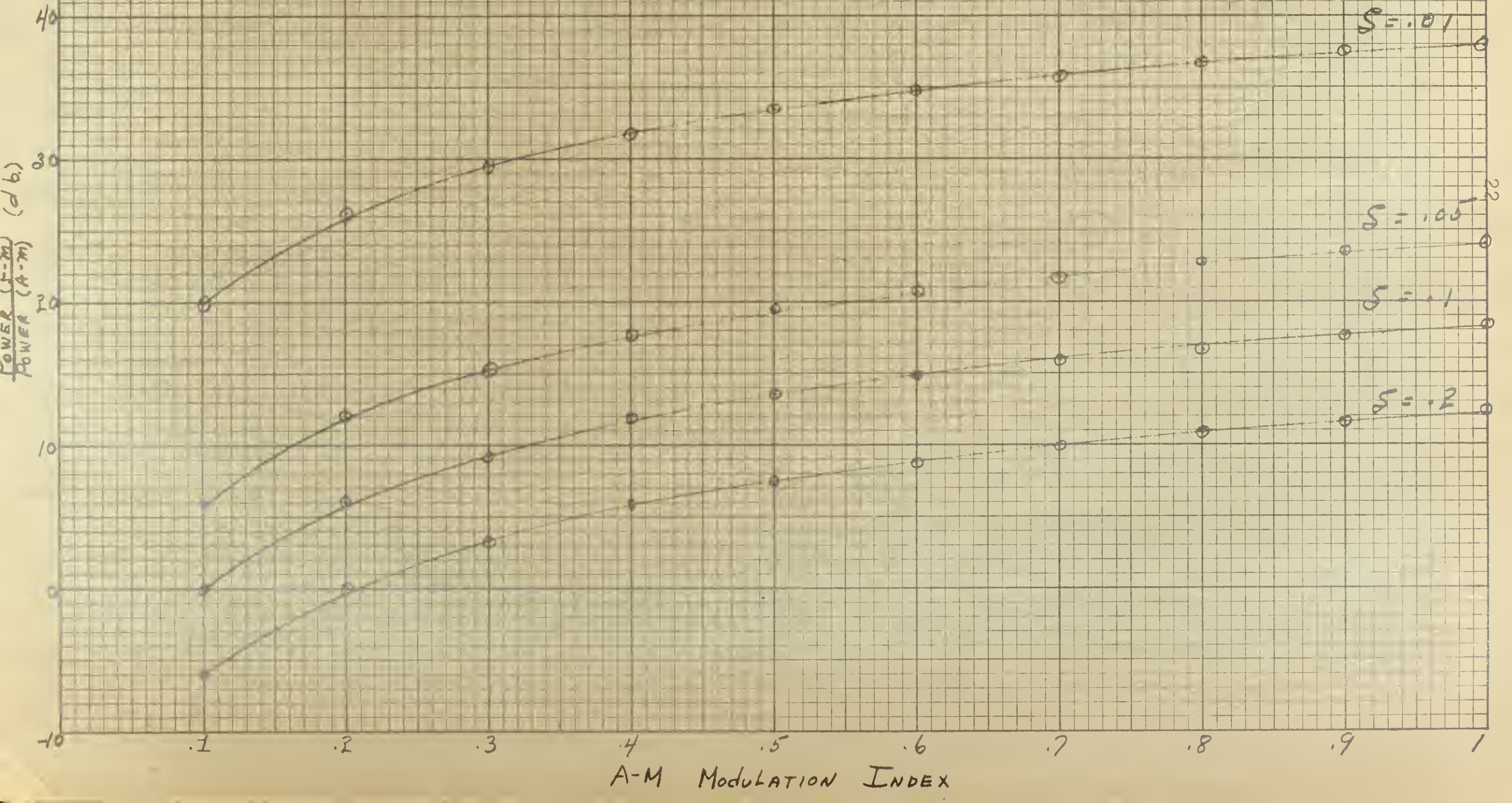
A brief discussion of the results is in order. The data signal power at the receiver output is determined by the post detector noise bandwidth and the $\frac{S}{N}$ ratio required at the servo phase meter input, whether the receiver input be an FM or AM signal. This in turn dictates the sideband power required at the receiver input. The curves can be

interpreted to show the amount of power that is not used by the receiver when an FM signal is used in lieu of an AM signal. This unused power must be generated by the transponder. The transponder is being called upon to produce more power than is required for overall system performance.

FIG. 3 COMPARISON OF DME GROUND STATION RECEIVER POWER INPUT REQUIREMENTS BETWEEN FM AND AM WHEN USING SYNCHRONOUS DETECTION

$\frac{\text{POWER (F-M)}}{\text{POWER (A-M)}}$ VS A-M MODULATION INDEX

WITH F-M MODULATION INDEX (S) AS A PARAMETER



B. Comparison of Ranging Capabilities when using FM or AM

Consider the transponder capable of producing a given power output, Pt, whether it be an FM or AM signal. The power intercepted by the ground station receiver is, signal power density x effective aperture of the antenna.

$$P_r = \frac{P_t}{4\pi r^2} A_e \tag{8}$$

(The transponder is considered a point source)

where r = transponder range

Ae = effective aperture of the ground station receiving antenna

Pr = received power at the ground station

For FM $$P_{t_{fm}} = \frac{P_{r_{fm}} 4\pi r_{fm}^2}{A_e} \tag{9}$$

For AM $$P_{t_{am}} = \frac{P_{r_{am}} 4\pi r_{am}^2}{A_e} \tag{10}$$

Since $P_{t_{fm}} = P_{t_{am}}$, it follows that

$$P_{r_{fm}} r_{fm}^2 = P_{r_{am}} r_{am}^2 \tag{11}$$

Now $P_{r_{fm}} = \frac{A^2}{2}$, $P_{r_{am}} = \frac{A'^2}{2} + \frac{m^2 A'^2}{4}$ subject to the constraint that $\delta A = m A'$

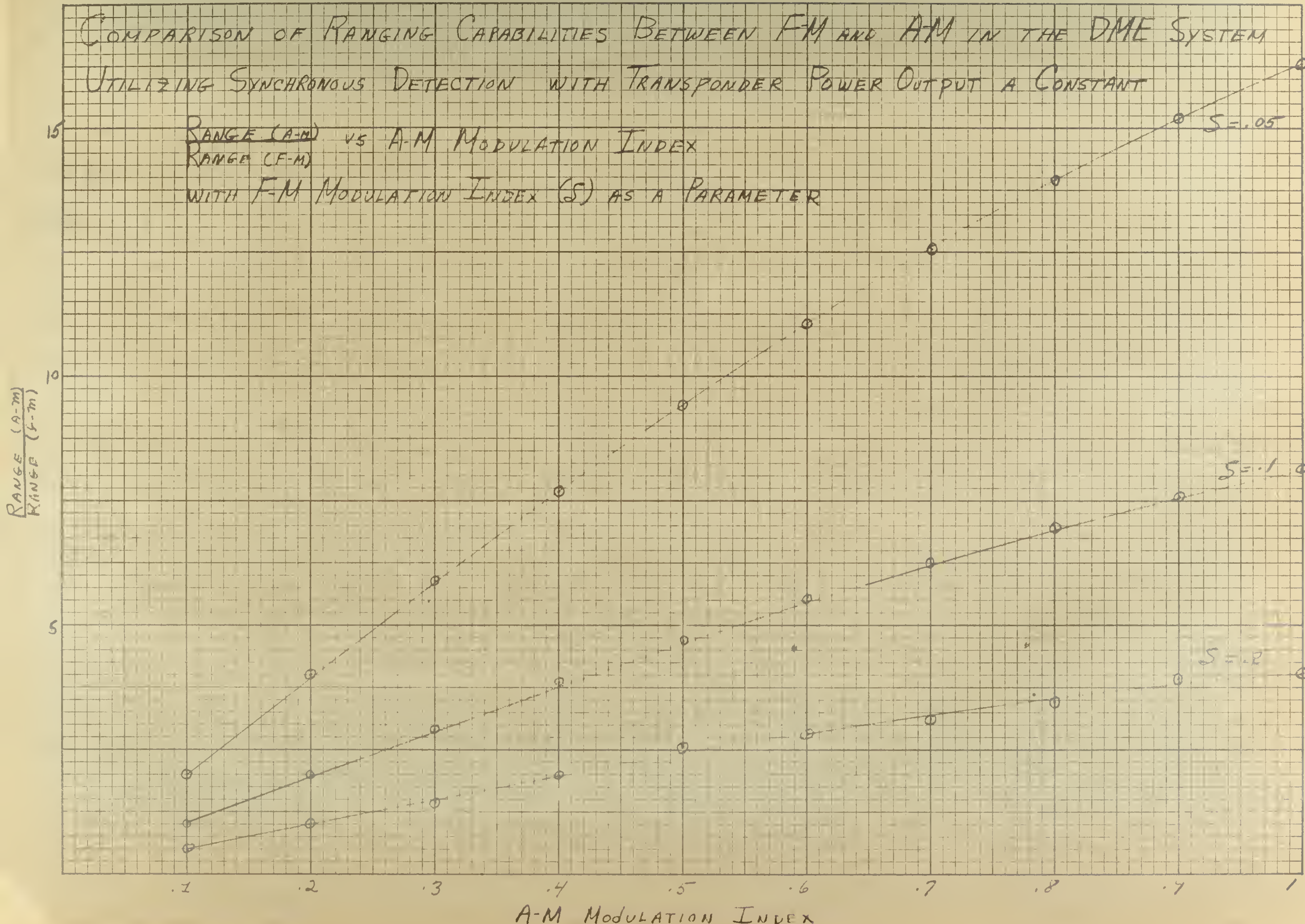
$$\frac{r_{am}^2}{r_{fm}^2} = \frac{P_{r_{fm}}}{P_{r_{am}}} = \frac{A^2}{A'^2 + \frac{m^2 A'^2}{2}} = \frac{m^2}{\delta^2 (1 + \frac{m^2}{2})} \tag{12}$$

$$\frac{r_{am}}{r_{fm}} = \frac{m}{\delta\sqrt{1 + \frac{m^2}{2}}} \tag{13}$$

A plot of $\frac{r_{am}}{r_{fm}}$ vs. m with δ a parameter is shown in Fig. 4

A brief discussion of the significance of these curves is also in order. At the ground station there is a receiver utilizing synchronous detection. The receiver has a fixed gain and a predetermined data level output. The input signal is either FM or AM. In either case the sideband

FIG. 4



COMPARISON OF RANGING CAPABILITIES BETWEEN F-M AND A-M IN THE DME SYSTEM
UTILIZING SYNCHRONOUS DETECTION WITH TRANSPONDER POWER OUTPUT A CONSTANT
RANGE (A-M) / RANGE (F-M) VS A-M MODULATION INDEX
WITH F-M MODULATION INDEX (S) AS A PARAMETER
S = .05
S = .1
S = .2
15
10
5
.1
.2
.3
.4
.5
.6
.7
.8
.9
1
A-M MODULATION INDEX
RANGE (A-M) / RANGE (F-M)

power is some minimum value determined by the post detector filter. The transponder total power output is the same for FM or AM. For an FM signal some maximum range, r, can be obtained. The factor by which r can be increased when an AM signal is used is the ordinate of the graph. Here, again it is shown that system performance is determined not by how much power is utilized but by how the power is distributed. Ranging capabilities are extremely sensitive to sideband power.

C. Synchronous Detector Action from $\frac{S}{N}$ ratio point of view when the input is FM or AM.

The analysis will be conducted thusly; for a given FM signal with power P and with additive white gaussian noise of N_0 watts/cycle band-limited to Wcps at the input to a synchronous detector, what is the modulation subtone output $\frac{S}{N}$ ratio; and, for a given AM signal with the same power P and with the same noise added to it, what is the modulation subtone output $\frac{S}{N}$ ratio. A comparison will be made between the two output $\frac{S}{N}$ ratios for both modulation techniques.

For the FM signal input, the low frequency multiplier output is $A \sin(\omega_c t + a(t)) \times C \cos \omega_c t$

$$= \frac{AC}{2} \sin a(t) \tag{14}$$

where $a(t) = \delta \sin \omega_m t$

With $\delta \ll 1$ $\quad \frac{AC}{2} \sin a(t) = \frac{AC\delta}{2} \sin \omega_m t \qquad (15)$

The signal power at the receiver output $\quad \frac{A^2 C^2 \delta^2}{8} \qquad (16)$

The noise power at the receiver output can be expressed as N_0

The AM signal may be written as

$$A'(1 + m \cos \omega_m t) \cos \omega_c t$$

$$= A' \cos \omega_c t + \frac{mA'}{2} \cos(\omega_c - \omega_m)t + \frac{mA'}{2} \cos(\omega_c + \omega_m)t \tag{17}$$

The average power $= \frac{A'^2}{2} + \frac{m^2A'^2}{4}$ (18)

This must equal the average power of the FM signal, therefore

$$\frac{A^2}{2} = \frac{A'^2}{2} + \frac{m^2A'^2}{4}$$

$$A' = \frac{A}{\sqrt{1+\frac{m^2}{2}}} \qquad (19)$$

The AM input signal may be written as

$$\frac{A}{\sqrt{1+\frac{m^2}{2}}}\left(1+m\cos\omega_m t\right)\cos\omega_c t \qquad (20)$$

When multiplied by $C\cos\omega_c t$ and considering only the output falling in the bandpass of the output filter, the signal output is then

$$\frac{CAm}{2\sqrt{1+\frac{m^2}{2}}}\cos\omega_m t \qquad (21)$$

The signal power output is $\frac{(CAm)^2}{8(1+\frac{m^2}{2})}$ (22)

The noise power output will be the same as in the FM case. Therefore, we may write the $\frac{S}{N}$ power ratio at the output as

$$\frac{S}{N} = \frac{\frac{(CAm)^2}{8(1+\frac{m^2}{2})}}{N} \qquad (23)$$

Comparing $\left.\frac{S}{N}\right|_{am}$ to $\left.\frac{S}{N}\right|_{fm}$ the following is obtained

$$\frac{\left.\frac{S}{N}\right|_{am}}{\left.\frac{S}{N}\right|_{fm}} = \frac{m^2}{(1+\frac{m^2}{2})\delta^2} \qquad (24)$$

Expressed in db, the improvement in AM over FM for equal input power is

Improvement (db) $= 10\log\frac{m^2}{(1+\frac{m^2}{2})\delta^2}$ (25)

Curves of this function are plotted in Fig. 5.

From consideration of these three analyses, the importance of

having the maximum amount of transponder transmitted energy at the sidebands of interest is readily seen. Its effects in so far as system ranging performance are very pronounced.

On the basis of what has been shown in Sections 4, 5 and 6, an AM system will be proposed as a solution to the problem stated in Section 2.

FIG. 5 Comparison of Sinusoidal Detection of AM and FM Signals with Equal Power Input to Detector

Curve S/N Improvement of AM and FM vs AM Modulation Index with FM Modulation Index as Parameter

7. Specific AM Methods and their Suitability for DME Use

Since AM will be proposed for use in the DME, an investigation of the specific types of AM and their applicability to the present problem will be discussed. Double sideband, single sideband, single sideband suppressed carrier and double sideband suppressed carrier will be treated.

The AM signal may be written

$$A(1 + a(t)) \cos \omega_c t \qquad (1)$$

where a(t) is the intelligence.

Let $a(t) = m \cos \omega_m t$

where m is the modulation index.

The received signal at the ground station receiver is

$$A \cos \omega_c (t-T) + \frac{Am}{2} \cos(\omega_c + \omega_m)(t-T) + \frac{Am}{2} \cos(\omega_c - \omega_m)(t-T), \qquad (2)$$

where T represents the time delay due to Transponder range.

The multiplier signal is $C \cos \omega_c t$. (3)

Double Sideband System

In the double sideband system equation (1) is multiplied by (3) to give as the low frequency output

$$\frac{AC}{2} \cos \omega_c T + \frac{AC}{2} \cos \omega_c T \, a(t-T). \qquad (4)$$

From this it can be seen that the desired intelligence is available. A d-c control voltage is also available for phase control of the reinserted carrier.

Single Sideband System

The signal may be represented as

$$A \cos \omega_c (t-T) + \frac{Am}{2} \cos(\omega_c + \omega_m)(t-T). \qquad (5)$$

The filtered output of the multiplier is

$$\frac{AC}{2}\cos\omega_c T + \frac{AmC}{2}\cos(\omega_m t - \omega_m T - \omega_c T). \tag{6}$$

If the carrier were suppressed the d-c term would not appear. The important factor in the single sideband system is that the phase desynchronization of the locally supplied carrier will appear directly as a phase error in the modulation subtone. Because of this, single sideband does not appear suitable for application to the DME System.

Double Sideband Suppressed Carrier

The signal may be represented as

$$\frac{Am}{2}\cos(\omega_c+\omega_m)(t-T) + \frac{Am}{2}\cos(\omega_c-\omega_m)(t-T). \tag{7}$$

The filtered output of the multiplier is

$$\frac{ACm}{2}\cos\omega_c T\cos\omega_m(t-T). \tag{8}$$

The information is available as in ordinary AM, however, there is no d-c control voltage to control the phase of the locally supplied carrier. This apparent disadvantage may be overcome by use of the circuit shown in Fig. 6

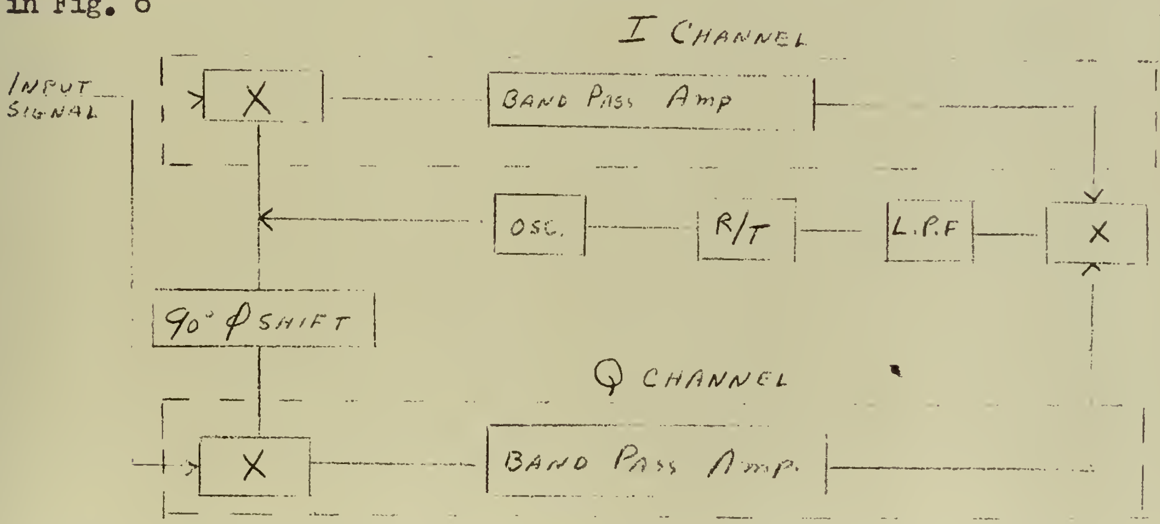


Fig. 6 - Double Sideband Suppressed Carrier Phase Control Circuit

The I channel output is $\frac{AC_c m}{2} \cos \omega_c T \cos \omega_m (t - T)$. (9)

The Q channel output is $\frac{AC_c m}{2} \sin \omega_c T \cos \omega_m (t - T)$. (10)

Multiplying equation (9) by (10) gives

$$\frac{1}{4}\left(\frac{AC_c m}{2}\right)^2 \sin 2\omega_c T. \tag{11}$$

This is a d-c voltage which can be applied to a reactance tube to control the locally supplied carrier phase.

The double sideband suppressed carrier system is truly an ideal system were it not for one important drawback. When noise is considered in the system, the low pass filter must have such a small passband in order to keep local oscillator phase jitter at a reasonable level, that the lock on time of the local oscillator will become enormous. This will become more lucid when local oscillator phase control is discussed. It can be seen from the diagram that a noise squared term will enter the low pass filter in addition to first power noise terms. From the convolution theorem it can be shown that this noise squared term produces the maximum noise spectral density at zero frequency. This scheme closely approaches a true autocorrelator for local oscillator phase control. For the above reasons the double sideband suppressed carrier system will not be proposed.

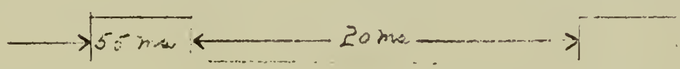
After consideration of the several approaches in the use of AM, it is felt that double sideband AM is the most desireable.

8. Obtaining the Synchronous Condition

It has been shown in preceeding analysis that in order to obtain the maximum benefit from synchronous detection, the reinserted carrier must be the same frequency and in phase with the carrier of the received signal. A method will be discussed for obtaining this required synchronism.

Before proceeding, it is desireable at this time to view some further details of operation of the DME equipment. To position the target, consider the case of obtaining three ranges. Three DME's are utilized. The method used to position a target is to key each DME for a specified interval of time, and obtain a range measurement during that time. Three sequential ranges may be combined to give a position.

The keying signal for any one DME appears as:



When considering maximum radial target acceleration of 10G, the carrier frequency (400 mcs.) can change approximately 260 cps in one second. Therefore, in one signal burst, the maximum carrier frequency change is 1.43 cycles. Between signal bursts the carrier frequency can change 5.2 cycles. This appears to define the operating conditions of a frequency or phase lock loop. However, the equipment must be capable of target acquisition. Under the most severe conditions, the doppler shift from nominal carrier frequency is approximately $\pm$ 24.12 kc. Therefore, the APC loop must be capable of locking on in a range $\pm$ 24.12 kc from the nominal carrier frequency. To be sure, the above mentioned conditions are a rare event, but their possibility must not be excluded. In order to ensure target acquisition, it will be necessary to have not only an automatic phase control loop but also an automatic frequency control loop. The automatic

frequency control loop will act to bring the locally supplied carrier within the range of the automatic phase control loop. After target acquisition, the automatic phase control loop will be capable of holding the locally supplied carrier in phase synchronization with the received carrier despite the gated nature of the transmitted signal.

For effective phase and frequency control of the locally supplied carrier, it is imperative that the received carrier signal have no sidebands within the expected range of variation due to doppler shift; otherwise, there would be a possibility of the locally supplied carrier locking on to one of the sidebands. If this happened, there would be no receiver output data signal.

For the present, however, it will be assumed that the carrier signal is available with no sideband signal within ± 25kc of the nominal value.

The Automatic Phase Control loop is really the heart of the entire synchronous detection scheme. Without this the entire system is void of any usefulness. Because of the importance of this circuit in its reliability and performance, it is paramount to consider some of the characteristics of operation, particularly under conditions of relatively high noise levels. There are two important operational parameters of the loop which require investigation. These are: under conditions of relatively poor signal to noise ratios what is the lock on time required, and, after lock on, what is the phase error or jitter due to the presence of noise in the system. Lock on time is important, since with each transmitter burst it is desired to have receiver output data. The oscillator must be capable of locking on in a short time compared to the 5.5 ms burst. Phase jitter of the locally supplied carrier after lock on is important since this will appear as amplitude jitter of the

receiver data signal. The data signal can be top and bottom limited to eliminate all amplitude jitter. The locally supplied carrier phase jitter should be limited to some conservative value, say 5° rms, in order that data signal limiting can be accomplished effectively.

Jaffe and Rechtin (4) have done considerable work in phase lock loops with success in correlating theoretical analysis with experimental data. The following method of analysis will be based upon their method.

The basic phase lock loop is shown in Fig. 7.

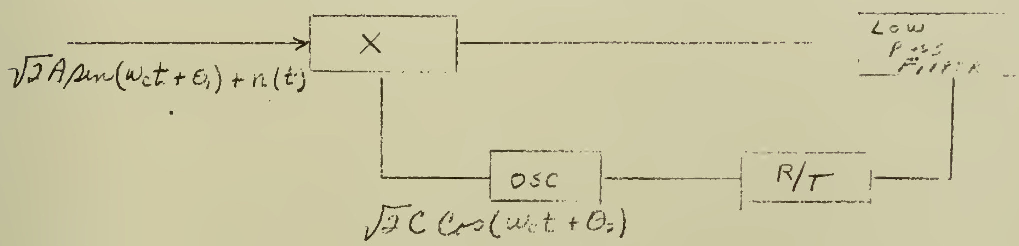
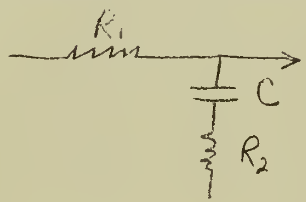


Fig. 7 - Typical phase lock loop

The low pass filter will be taken as the conventional APC filter



$$R_2 << R_1$$
$$\tau_1 = R_1 C \qquad \tau_2 = R_2 C$$
$$R_2 = mR_1 \qquad \tau_2 = m\tau_1$$

with Laplace transfer function $F(s) = \dfrac{\tau_2 s+1}{(\tau_1+\tau_2)s+1} \approx \dfrac{\tau_2 s+1}{\tau_1 s+1}$.

The loop input signal $= \sqrt{2}A\sin(\omega_c t+\theta_1) + n(t)$

where A = rms value of carrier

n(t) = white gaussian noise bandlimited to Wcps with spectral density N_o watts/cycle centered about fc.

C = rms value of locally supplied carrier.

The low frequency output of the multiplier is

$$AC\sin(\theta_1-\theta_2) + n(t)C\sqrt{2}\cos(\omega_c t+\theta_2). \quad (1)$$

By the convolution theorem

$$n(t)\sqrt{2}\,C\cos(\omega_c t+\theta_2) = Cn(t) \quad (2)$$

where Cn(t) may be considered as centered about 0 frequency.

The multiplier output is then

$$C[A\sin(\theta_1-\theta_2) + n(t)]. \quad (3)$$

Since the local oscillator is phase locked the following approximation may be made

$$\sin(\theta_1-\theta_2) \doteq \theta_1-\theta_2. \quad (4)$$

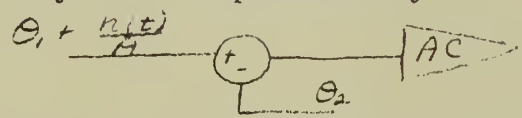
The multiplier output is now

$$C[A(\theta_1-\theta_2) + n(t)]. \quad (5)$$

Dividing and multiplying by A gives

$$AC\left[(\theta_1-\theta_2) + \frac{n(t)}{A}\right]. \quad (6)$$

The multiplier may now be represented by



In order to maintain the representation of the multiplier consistent, a constant, Km, must be introduced, in order to relate the multiplier output voltage to the amplitude of the two inputs and their phase difference. Km has the units $\frac{1}{\text{rad. volt}}$.

The local oscillator and reactance tube combination produces an output signal whose frequency is proportional to the voltage impressed upon it. The output phase is then the integral of the input voltage.

This may be represented in Laplace transform notation as

$$\xleftarrow{\ \theta_2\ } \boxed{K/s} \longleftarrow$$

where Kv has the units $\frac{\text{rdn/sec.}}{\text{volt}}$.

The entire loop may be represented as shown in Fig. 8 (using Laplace notation).

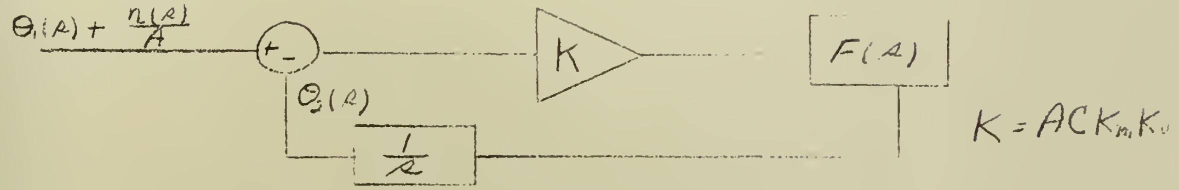


Fig. 8 - Linear approximation of phase lock loop

Let the closed loop transfer function = Y(s).

$$\theta_2(s) = Y(s)\,\theta_1(s)$$

$$= K\left[\theta_1(s) - \theta_2(s)\right]\frac{F(s)}{s}$$

$$\theta_2(s)\left[1 + \frac{KF(s)}{s}\right] = \frac{KF(s)}{s}\,\theta_1(s)$$

$$\theta_2(s) = \frac{KF(s)}{s + KF(s)}\,\theta_1(s)$$

$$Y(s) = \frac{KF(s)}{s + KF(s)}. \qquad (7)$$

It is easily shown that the phase noise power, σ_n^2, at the output of the reactance tube controlled oscillator is

$$\sigma_n^2 = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} |Y(s)|^2\,\theta_n(0)\,ds. \qquad (8)$$

$\theta_n(0)$ = the input phase noise power spectral density

$$\theta_n(0) = \frac{N_0}{A^2} = \left(\frac{N}{A^2}\right)\frac{1}{2\Delta f}$$

where Δf is the input noise bandwidth and N is the total noise power in that bandwidth.

$$|Y(s)|^2 = Y(s)\,Y(-s)$$

$$= \frac{K^2\left\{\frac{\tau_2 s + 1}{\tau_1 s + 1}\right\}\left\{\frac{1 - \tau_2 s}{1 - \tau_1 s}\right\}}{\left[s + \frac{K(\tau_2 s + 1)}{(\tau_1 s + 1)}\right]\left[-s + \frac{K(1 - \tau_2 s)}{(1 - \tau_1 s)}\right]}$$

$$|Y(s)|^2 = \frac{-K^2\{\tau_2^2 s^2 - 1\}}{(\tau_1 s^2 + (1+K\tau_2)s + K)(\tau_1 s^2 - (1+K\tau_2)s + K)}$$

$$\sigma_n^2 = \frac{-K^2 N_0}{2A^2}\,\frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \frac{|Y(s)|^2}{-K^2}\,ds. \qquad (9)$$

Integrals of this form have been solved by contour integration and are conveniently tabulated.[1]

$$\sigma_n^2 = \left(\frac{N}{2A^2\Delta f}\right)\left(\frac{K}{2}\right)\left(\frac{m^2\tau_1 K + 1}{1 + mK\tau_1}\right). \qquad (10)$$

It can be shown (6) and (7) that the time required for lock on is given by

$$T = 5\left[\frac{(\Delta f_c)^2}{(f_n)^3}\right] \qquad (11)$$

where Δf_c is the initial frequency difference between the received carrier and the locally supplied carrier and fn is the phase loop effective noise bandwidth with the suppositions that $(1+m\tau_1 K)^2 = 4\tau_1 K$, $mK\tau_1 >> 1$.

It has been found that these suppositions produce optimum phase lock performance.

It can also be shown that $2f_n = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} |Y(s)|^2\,ds.$ (12)

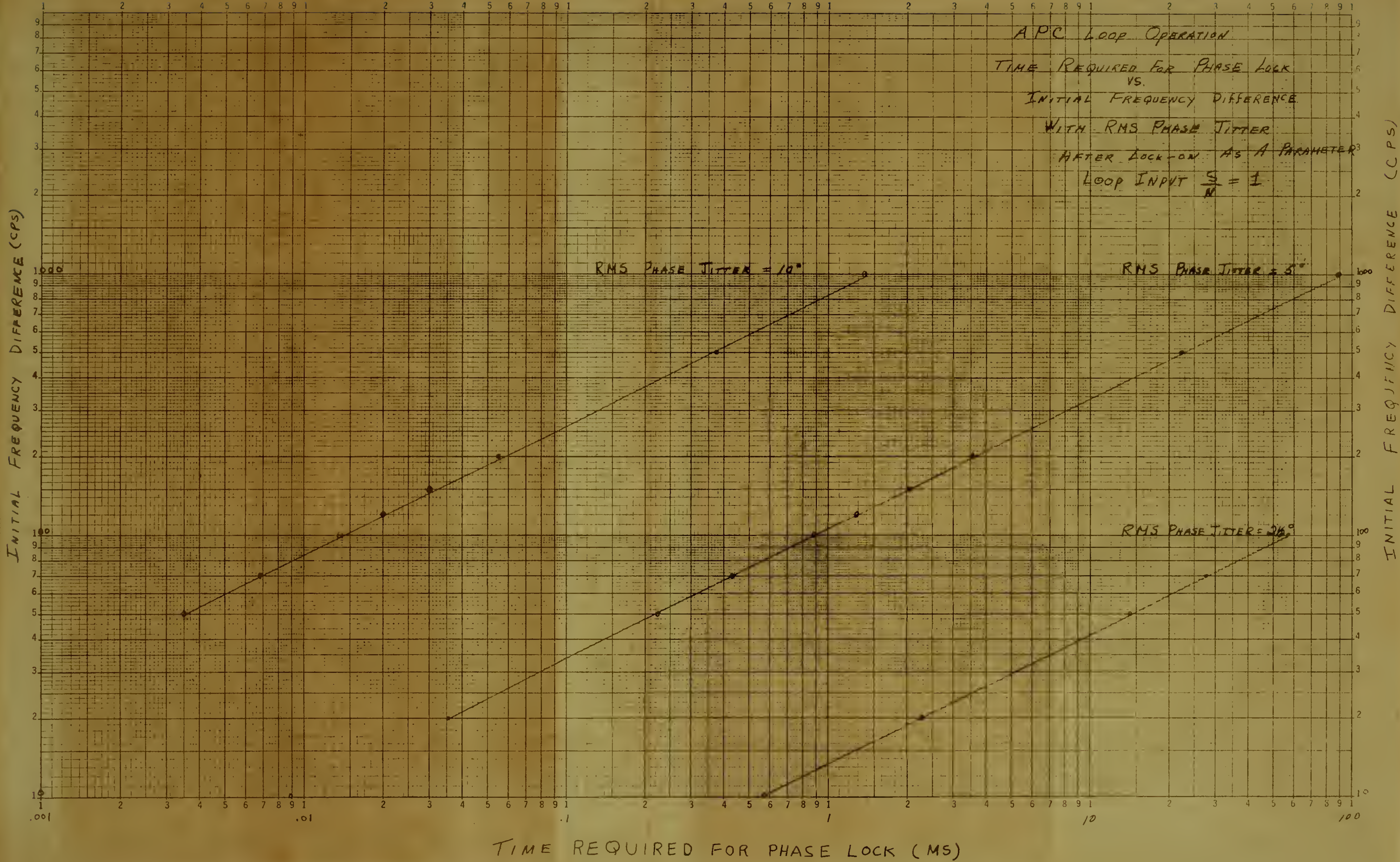
Substituting equation (12) in (10) gives

$$\sigma_n^2 = \left(\frac{N}{2A^2\Delta f}\right)(2f_n). \qquad (13)$$

Using equations (11) and (13), curves of initial oscillator frequency difference from the received carrier vs. time required for lock on with an input $\frac{S}{N}$ ratio of 1 are shown in Fig. 9.

[1] James, Nichols, Phillips, Theory of Servomechanisms, M.I.T. Rad. Lab. Series, Vol. 25, pp. 369-370.

FIG. 9


APC Loop Operation
Time Required For Phase Lock vs. Initial Frequency Difference With RMS Phase Jitter After Lock-on As A Parameter
Loop Input S/N = 1
RMS Phase Jitter = 10°
RMS Phase Jitter = 5°
RMS Phase Jitter = 2.5°
Initial Frequency Difference (CPS)
Time Required for Phase Lock (MS)
.001
.01
.1
1
10
100
1000

The noise bandwidth has been taken as 50 kc since the doppler shift may be ± 25 kc from the nominal carrier frequency.

The curves show the pronounced effect of averaging time on time required for lock on. The longer the averaging time, the less phase noise jitter, all at the expense of increased time for phase lock. Operation on the 5° rms phase jitter curve is recommended for the DME system. This appears to give the best compromise between frequency differences that may occur during signal bursts, lock on time and resultant data amplitude jitter which can be effectively removed by limiting. As $\frac{S}{N}$ ratios become increasingly better, resultant data amplitude jitter will decrease.

The curves also show that some auxiliary control is required for initial lock on in target acquisition. A swept oscillator arrangement may be employed or a circuit similar to that developed by Richman (7) may be utilized.

In the previous discussion, it has been assumed that the carrier was available with no sidebands within the expected carrier shift due to doppler. Since the lowest modulation frequency is 192 cps a single AM scheme cannot be used. A solution to the problem is obtained by modulating one of the modulation subtones by the other subtones and using this signal to modulate a main carrier.

Let f_1 = .192 cps

f_2 = 1.9 kc

f_3 = 7.9 kc

f_4 = 61 kc

f_5 = 491 kc

f_c = main carrier at 430 mcs. (Interrogator Transmitter carrier)

Let f_1, f_2 and f_3 modulate f_4 in a balanced modulator. The spectrum at the output of the balanced modulator will contain components at $f_4 \pm f_1$, $f_4 \pm f_2$, $f_4 \pm f_3$. To this add f_4 and f_5. Let this resultant signal amplitude modulate f_c. The transmitter spectrum will consist of $f_c \pm f_4$, $f_c \pm f_5$, $f_c \pm f_4 \pm f_1$, $f_c \pm f_4 \pm f_2$, $f_c \pm f_4 \pm f_3$, f_c.

The nearest sidebands to f_c are now $f_c + f_4 - f_1 - f_2 - f_3$ and $f_c - f_4 + f_1 + f_2 + f_3$. The main carrier has no frequency component within approximately 50 kc on either side, and is now readily available for use in a phase lock circuit. It is to be noted that if double sideband suppressed carrier were used there would be no necessity for a double modulation scheme. The sidebands due to the 491.76 kc signal could be used in the phase control loop shown in Fig. 6. Since there has been double modulation, there must be double detection at the receiver. The second detection is relatively simple since the 61 kc signal is already available at the receiver for a second synchronous detection.

9. Proposed system and comparison with the present system.

All of the elements required for a block diagram of the proposed DME system have heretofore been presented. Shown in Fig. 10 is a block diagram of the interrogator transmitter. It consists essentially of double sideband suppressed carrier amplitude modulation followed by amplitude modulation of a main carrier. The first modulation need not be suppressed carrier, however, by use of the balanced modulator and adder combination, the amplitude of all of the resultant spectral components can easily be adjusted to that required by the receiver post detection filters for a $\frac{S}{N}$ ratio of 10.

In Fig. 10, f_1 = .192 kc $\quad$ f_4 = 6147 kc
f_2 = 1.9 kc $\quad$ f_5 = 471 96 kc
f_3 = 71 kc

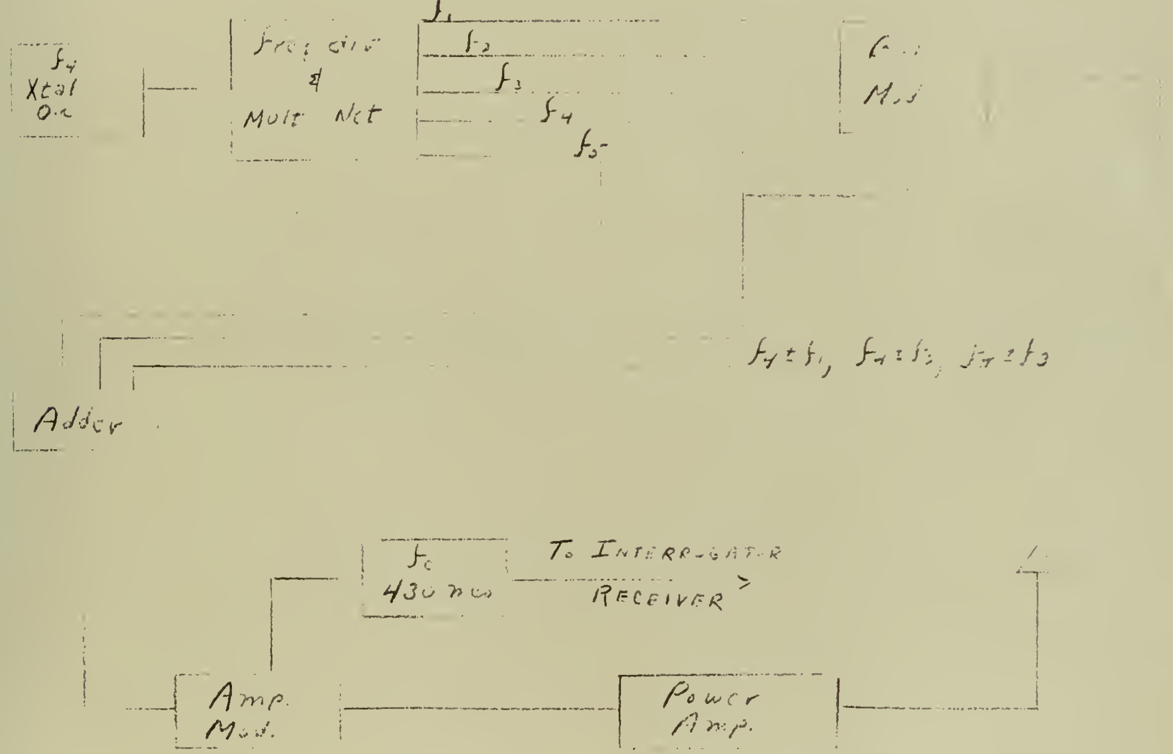


Fig. 10 - Block diagram of proposed interrogator transmitter

In Fig. 10, the transmitter output may be expressed as

$$A[1 + g(t)] \cos \omega_c t. \tag{1}$$

where $g(t) = a(t)\,b(t) + b(t) + c(t)$

and $a(t) = \cos \omega_1 t + \cos \omega_2 t + \cos \omega_3 t$

$b(t) = \cos \omega_4 t$

$c(t) = \cos \omega_5 t.$

The requirements of the transponder are merely a spectral shift and amplification. It is the opinion of this author that the transponder should be as simple as technically possible, performing only that which is essentially required. With this in mind, the block diagram in Fig. 11 is proposed. Cubic Corporation has had this transponder scheme under

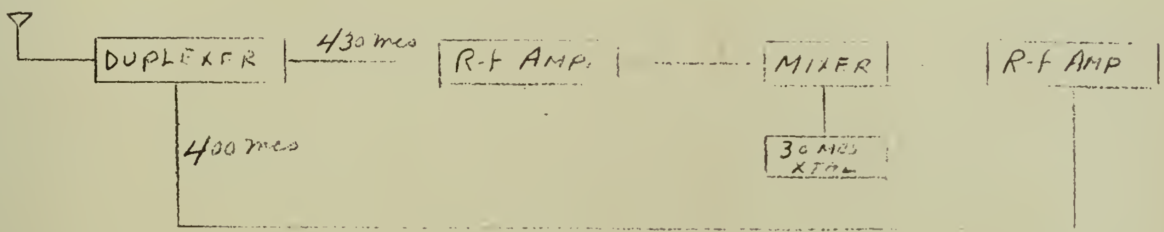


Fig. 11 - Block Diagram of Proposed Transponder

study and ultimate utilization appears favorable. The present transponder utilizes detection of the received signal and remodulation of another carrier. The method of Fig. 11 appears to reduce the necessary hardware required. There will be a time delay through the system, but it is planned to have this a known constant, and any variations from this to have negligible effect on the overall system. The constant time delay through the transponder can be calibrated out of the system. It will be shown that the radiated power required from the transponder will be of the order of 300 - 400 milliwatts. The gain of the r-f amplifier can be of the order of 50 db., giving an overall transponder gain of approximately

100 db. Amplifiers of this nature operating in the UHF region with bandwidths of from 10 to 30 mcs have been built and tested (5). The band-pass is broad enough to ensure phase stability through the system. 30 mcs crystal oscillators are available with stabilities in the order of one part in 10^6.

The interrogator receiver shown in Fig. 12 is the most complicated part of the system. In addition to amplification stages it requires one mixing stage and two detection stages. The local oscillator for the mixer is taken directly from the main carrier generated in the interrogator transmitter. No afc control is required. The frequency will always be 30 mc $\pm$ doppler $\pm$ any error in the 30 mc local oscillator in the transponder. The latter error will be negligible hence for all practical purposes the i-f frequency will always be 30 mcs $\pm$ doppler. The r-f amplifier can be very similar to the r-f amplifier in the transponder. The i-f amplifier can have a modest gain of approximately 30 db. The majority of the receiver overall gain can be accomplished in the individual data channels, since in general it is easier to amplify one frequency than it is to amplify a band of frequencies.

Upon inspection of the block diagram in Fig. 12, it can be seen that in the i-f amplifier the signal will not be 30 mcs but 30 mcs $\pm$ doppler since there is no afc control of the 430 mcs mixing signal. A detuning effect in the i-f strip is inherent in the system. What, then, is the effect of detuning upon ranging accuracy. Several reasonable assumptions must be made first. It can be assumed that the i-f strip can have an overall transfer characteristic such that the bandwidth is 4 mcs at the 3 db. points and that the phase slope is linear out to slightly beyond the farthest sideband, approximately $\frac{1}{2}$ mc away from the center frequency.

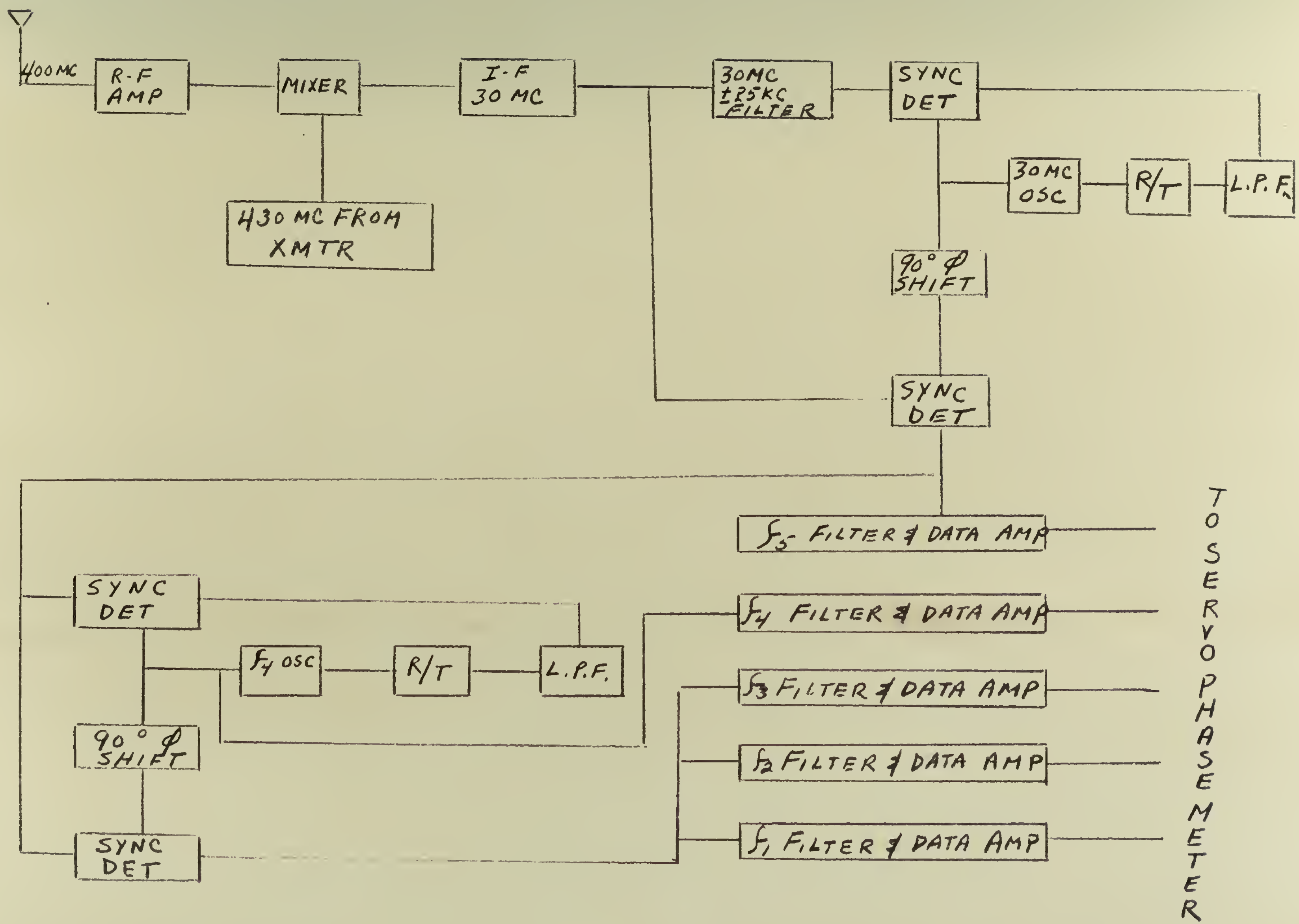


Fig. 12 - Block Diagram of Proposed Interrogator Receiver

It is even reasonable to consider the phase slope linear to the 3 db points, however, this need be assumed only in order to obtain the slope of the phase line around the center frequency. The phase slope can be taken as $45^\circ/2\ mc$. The amplitude characteristics may be assumed constant over the region of interest. There will be a slight amplitude reduction factor but this is negligible. With the preceeding assumptions the input signal will arrive undistorted at the output with some delay, T.

Consider the AM signal arriving at the output of the perfectly tuned strip as (amplitude Factors will be neglected)

$$\cos \omega_c t + \tfrac{1}{2}\cos(\omega_c - \omega_m)t + \tfrac{1}{2}\cos(\omega_c + \omega_m)t. \tag{1}$$

For any mistuning the output signal may be written

$$\cos \omega_c (t - T') + \tfrac{1}{2}\cos(\omega_c - \omega_m)(t - T') + \tfrac{1}{2}\cos(\omega_c + \omega_m)(t - T'). \tag{2}$$

where T' is any additional delay due to mistuning.

In the synchronous detector, the locally supplied carrier will be locked with $\cos \omega_c (t - T')$. Multiplying equation (2) by $\cos \omega_c (t - T')$ and neglecting terms at 2 wct, the multiplier output is

$$\tfrac{1}{2}\cos \omega_m (t - T'). \tag{3}$$

Now WmT' represents a phase error in addition to that which would be encountered in the perfectly tuned i-f strip. This phase error is obtained directly from the phase slope. The phase error is given by the phase slope times the cycles detuning. For maximum doppler shift this phase error, ϕ_e, is

$$\phi_e = \frac{45^\circ}{2 \times 10^6} \times 25 \times 10^3 = .562^\circ \tag{4}$$

This amounts to an error of approximately 1.6 ft. in range determination. This is considered tolerable and not worth the expense of incorporating

elaborate circuits to maintain the i-f frequency precisely at 30 mcs. A .3 mcs detuning would cause an error in range measurement of about 18 ft. A 300 kc detuning effect is quite large. A broad band strip is most desireable from a system accuracy point of view.

It is planned to obtain a radial velocity vector through measurement of doppler shift which will be present in the 30 mcs locked oscillator. A constant error will result due to the fact that the interrogator transmits at 430 mcs and the transponder transmits at 400 mcs. This may be calibrated out of the system. There is an inherent error in the system due to the instability of the 30 mcs oscillator in the transponder. Assuming stabilities of 1 part in 10^6 a maximum error of 30 cps can be present in the doppler shift. Using the conventional doppler shift equation in the form $f_d = \frac{89.5 v}{\lambda}$ where v is velocity in miles per hour and λ is the carrier wavelength in centimeters, the maximum velocity error due to the 30 cps error is approximately 40 ft/sec. At high velocities, say 20,000 ft/sec this error is only .2%, however at very low velocities it is a sizeable percentage. This may or may not be suitable for some missile applications. It is not entirely impossible that crystal oscillators are available with stabilities of the order of magnitude of 1 part in 10^7. This would reduce the error to 4 ft/sec. This error is inherent in the system and does not include any measuring errors. Admittedly this is not the ultimate to be desired in velocity measurements, however it is instantaneous and appears to be a step in the right direction. If substantially greater accuracy is required, greater circuit complexity must be realized in the transponder. But since the proposed transponder circuitry has been reduced from what is presently employed, the greater complexity may not be

objectionable. A stage of regenerative frequency division and frequency multiplication may be used in lieu of mixing. This would negate any inherent variable errors in the system.

It will be interesting to compare the present system with the proposed system. The interrogator transmitter may be slightly more complex in the proposed system. The transponder has been greatly reduced in circuitry, although a high precision crystal oscillator is demanded. The interrogator receiver has been increased in complexity. Overall, the system is considered more complex and higher in cost than the present system, but not objectionably so. The primary advantage of the proposed system is the net reduction of power output required from the transponder in order to obtain 200 mile range. This has further implications which will be discussed forthwith.

First, consider the reduction of transponder power output which is attainable due to synchronous detection. The interrogator receiver noise power spectral density generated is 4×10^{-20} watts/cycle (2). The carrier signal power required for effective phase lock operation (referring to Section 8) is given by

$$\frac{S}{(4 \times 10^{-20})\Delta f} = 1$$

where Δf = range of expected doppler shift, $\pm$ 25 kc

$$S = 20 \times 10^{-16}$$

The data signal power required may be calculated from a knowledge of the receiver output $\frac{S}{N}$ ratio required and an estimate of probable obtainable post detection filters. The data channels and post detection filter

bandwidth estimates are tabulated below.

	Data Channel	Post Detection Filter Bandwidth (effective noise bandwidth)
1.	491 kc	1 kc (obtainable with crystal filter)
2.	61 kc	1 kc " " " "
3.	7.9 kc	1 kc
4.	1.9 kc	.2 kc
5.	.19 kc	.1 kc

For all channels an output $\frac{S}{N}$ ratio of 10 is desired. From this the sideband power is calculated and tabulated below.

	Data Channel	Sideband Power in Watts (one sideband)
1.	491 kc	4×10^{-16}
2.	61 kc	4×10^{-16}
3.	7.9 kc	$4 \times 10^{-16} \times 2 = 8 \times 10^{-16}$
4.	1.9 kc	$.8 \times 10^{-16} \times 2 = 1.6 \times 10^{-16}$
5.	.19 kc	$.4 \times 10^{-16} \times 2 = .8 \times 10^{-16}$

The factor of 2 is required for the 7.9 kc, 1.9 kc and .19 kc channels since these signals have modulated the 61 kc signal. The total sideband power, upper and lower, is the summation of the individual powers multiplied by 2. This gives 36.8×10^{-16} watts. The total signal power is required at the input to the r-f amplifier is the carrier power plus the sideband power, 56.8×10^{-16} watts. Allowing 3 db attenuation between the antenna and the input of the r-f amplifier, the total signal power required at the interrogator receiver antenna is 113.6×10^{-16} watts.

The transponder may be considered as a point source, therefore the power received by the interrogator is

$$P_r = \frac{P_t}{4\pi r^2} A_e \qquad (1)$$

where A_e = receiving antenna effective aperture

r = transponder range

P_t = transponder output power

The effective aperture used for the interrogator receiving antenna is .0795 λ^2 where λ^2 = 6.05 ft, corresponding to a frequency of 400 mc. Considering a range of 200 miles and substituting the known quantities in equation (1), the power required at the transponder output is 334 milliwatts. This is radiated power. The transponder radiated power presently used is 35 watts. The proposed system can realize a net reduction of 20.2 db in transponder power output. This represents a substantial increase in system performance and is a primary advantage of the proposed system.

This large reduction of output power required suggests the possibility of transistorizing the transponder circuit. It is a field of worthwhile investigation. A further implication is the facility with which ranging capabilities may be increased. To double the range, approximately 1.2 watts will be needed, to increase the range by a factor of 5 only 8.3 watts are required. Whereas with the present system to increase the range by a factor of 5, 675 watts must be radiated from the transponder.

10. The Optimum Receiver

As mentioned in Section 3, the method used in arriving at the solution to the formulated problem was essentially a heuristic approach, that is a solution was guessed and found to be satisfactory. But is this the best solution? Furthermore, if FM must be used, what is the optimum receiver? The proposed solution certainly does not lend itself very well to FM. The answers to these questions require a more philosiphical approach. The typical receiver is presented with a disturbance which is either noise alone or signal and noise. For strong signals noise can be neglected and a third category could be spoken of - signal alone. But this is of inconsequential interest here. The final object of any receiver is either to operate on the disturbance and make a decision as to whether the disturbance came from noise alone or signal and noise; or, after having made the decision that the disturbance came from signal and noise, give indications of one or all of the signal parameters. That is, a receiver is a device that merely detects the presence of a signal; or, a device that not only detects the presence of a signal but also extracts signal parameters. This paper has been concerned with the latter type receiver.

A logical approach to receiver design is from a causality point of view. It is epistomologically true that every effect has a cause (the converse is not necessarily true) and from a knowledge of an effect, a certain amount of information can be determined about the cause. Every receiver effectively operates on a disturbance effect at its input and determines some information about the cause of this disturbance. The principle of causality has been used by Woodward and Davies (10) (11) in their investigation of receiver design.

The nature of the disturbance at the receiver input is statistical. Therefore it is quite logical to use statistical tools in the application of causality. The specific method used by Woodward and Davies is "a posteriori probability". The apriori probability of the signal occuring must also be known. The receiver effectively looks at the disturbance, is given a probability of a signal being present, and then produces the probability that the signal was actually present. Before reception there is a probability of occurrence of all possible messages. After reception it is desired that one particular message will be chosen as the correct one. The apriori probability is changed into a posteriori probability.

The above may be stated more precisely from a consideration of some elements of probability theory. The probability that two events will occur simultaneously is given by

$$P(x,y) = P(x)\,P(y/x) = P(y)\,P(x/y) \tag{1}$$

where P(x,y) is the probability of joint occurrence of events x and y

P(x) is the probability of occurrence of event x

P(y) is the probability of occurrence of event y

P(x/y) is the probability of occurrence of event x given event y

P(y/x) is the probability of occurrence of event y given x

Equation (1) may be considered as discrete probabilities or probability density functions. For application to signal detection, let x = the transmitted signal and let y = the received signal at the receiver input. From equation (1)

$$P(x/y) = \frac{P(x)\,P(y/x)}{P(y)} . \tag{2}$$

P(y) is a constant for any received signal and is of such a value that $\int_{-\infty}^{\infty} P(x/y)\,dx = 1$. Therefore (2) may be written

$$P(x/y) = k\,p(x)\,P(y/x). \tag{3}$$

When considering x as the modulating intelligence a(t) and y the received signal, it can be shown (5) that equation (3) reduces to

$$P(a(t)/y) = k\,P(a(t))\,p(n) \tag{4}$$

where p(n) is the probability density function of the noise.

An ideal receiver would be one which calculated the probabilities of all possible messages when given an input. This is not very practical however. A logical approach is to pick a message which maximizes the probability of a(t) for a given receiver input. An optimum receiver would be one which produced the most likely a(t) at its output after having been given an input y. With this in mind, Thomas (8) has derived equations which indicate the form of an optimum receiver for an AM signal and an FM signal with a(t) a noise like modulating voltage independent of the actual noise. Based on this his results show that the optimum receiver for the AM case is the synchronous detector followed by a Weiner type filter. For the FM case wherein the approximation that sin a(t) = a(t) is made the optimum receiver is essentially the same as in the AM case. For wideband FM the optimum receiver takes the form of a locked oscillator. A complete analysis of the FM optimum receiver has not as yet been made due to the difficulty in dealing with the nonlinear nature of the feed-back loop.

11. Conclusions and Recommendations

This paper has presented a solution to the two-fold problem as put forth in Section 2. The use of synchronous detection and amplitude modulation is the basis of the solution. The primary advantage of the proposed system lies in the relatively high transponder power output reduction that can be realized. The power level can be reduced to the level which may be readily adaptable to transistor circuitry. Of even greater importance is the facility with which ranging capabilities can be increased in so far as power levels are concerned. Ranging in the order of magnitude of 1000 miles can be obtained with a matter of watts at the transponder transmitter.

Velocity information is available although the accuracy may not be as great as desireable. This is not an insurmountable problem. The solution exists in making the spectral shift in the transponder more accurate. This may be accomplished precisely, as mentioned in Section 9, through the use of frequency division and multiplication networks in lieu of conventional mixing.

The importance of phase synchronization has been stressed and certain design parameters of interest have been recommended for negligible data signal distortion due to noise.

The philosophy taken by the author has been to put the burden of complexity at the interrogator leaving the transponder as simple as possible. This is in the interest of system reliability. Coherent detection could be performed at the transponder resulting in a power reduction at the interrogator transmitter, but it is felt that it is simpler to generate the additional power required at the interrogator than it is to install complex circuitry in the transponder. The

transponder to interrogator link has been considered the sensitive link.

The overall system will require additional circuit complexity and a higher cost but it is not prohibitive in light of the advantages gained.

It is recognized that this system may not be the optimum physically possible merely from a signal detection point of view. However, it may be stated that it is an advance in the proper direction.

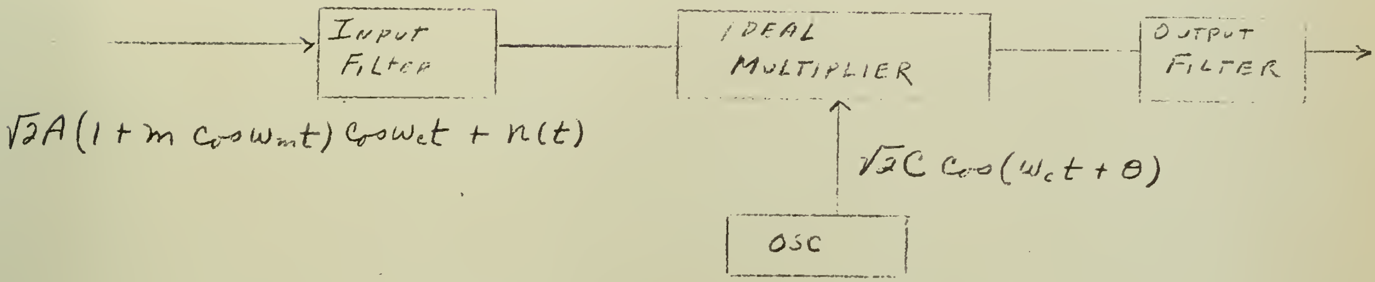
There are several recommendations which may be made for further investigation. As mentioned in Section 7, it would be theoretically possible to obtain the required phase-locked carrier by suitable operations on the sidebands. Use of this method for obtaining the required carrier would eliminate the necessity of a double modulation and detection scheme. The author has not at this time completed an adequate analysis of this method of obtaining carrier phase lock. Further investigation is recommended to properly determine suitability for use in the DME.

There is a need for further investigation into the optimum demodulator for a wideband FM system. The synchronous detector as offered here does not appear to be particularly applicable to FM. This investigation is of extreme importance if FM is demanded in the DME system as a part of a larger and more complex system.

It may be noted that this paper has spoken of AM and FM exclusively. It is not intended that this is the only way of obtaining accurate ranging information. There may be possibilities of performing the desired functions of the DME by means of pulse code techniques, and these techniques may prove more effective than ordinary AM and FM. These possibilities may be exploited with favorable results.

APPENDIX I

ANALYSIS OF THE BASIC SYNCHRONOUS DETECTOR



The input filter has a transfer function

$$|H_i(f)| = \frac{1}{\sqrt{1 + \left(\frac{f - f_c}{\alpha}\right)^2}} \qquad f \geq 0$$

$$= \frac{1}{\sqrt{1 + \left(\frac{f + f_c}{\alpha}\right)^2}} \qquad f \leq 0$$

fc = input filter center frequency

α = $\frac{1}{2}$ power bandwidth

$\alpha <<< f_c$

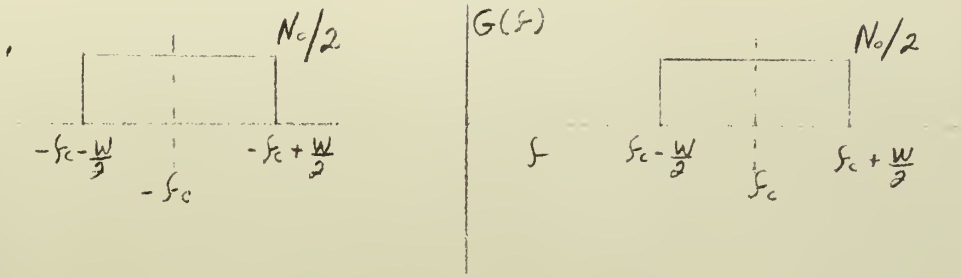
The output bandpass filter has a transfer function

$$|H_o(f)| = \frac{1}{\sqrt{1 + \left(\frac{f - f_m}{\beta}\right)^2}} \qquad f \geq 0$$

$$= \frac{1}{\sqrt{1 + \left(\frac{f + f_m}{\beta}\right)^2}} \qquad f \leq 0$$

The input noise, n(t), is assumed to be additive white gaussian noise, centered about fc and band-limited to W cycles with power spectral

density of No watts/cycle. The spectrum of the noise is thus:



It is assumed that the intelligence part of the received signal s(t) is completely passed by the input filter unattenuated. The noise input, n(t), will pass undisturbed to the multiplier if W is sufficiently small. Consider $W > \alpha$. This will perturb n(t) in some manner and can be expressed as n'(t). The input to the multiplier is then s(t) + n'(t).

The multiplier output is then

$$e_m(t) = \sqrt{2}\, C \cos(\omega_c t + \theta)\left[s(t) + n'(t)\right]$$

$$= s(t)\sqrt{2}\, C\cos(\omega_c t + \theta) + n'(t)\sqrt{2}\, C \cos(\omega_c t + \theta)$$

$$= \sqrt{2}A(1 + m\cos\omega_m t)\cos\omega_c t \times \sqrt{2}\, C\cos(\omega_c t + \theta)$$

$$+ n'(t)\sqrt{2}\, C\cos(\omega_c t + \theta)$$

Terms centered about $2\omega_c$ will be neglected, therefore

$$e_m(t) = \frac{AC}{2}\cos\theta + \cos\theta\frac{mAC}{2}\cos\omega_m t + n'(t)\sqrt{2}\, C\cos(\omega_c t + \theta)$$

The signal out of the bandpass filter will be $\cos\theta\,\frac{mAC}{2}\cos\omega_m t$ with signal power $= \frac{1}{2}\left(\frac{mAC\cos\theta}{2}\right)^2$

It is desired to find the noise power at the output of the bandpass filter. This can be found from $\phi_{nn}(\tau)$ for $\tau = 0$, or by integrating the noise power spectral density at the output of the filter.

$$\text{Noise power} = \int_{-\infty}^{\infty} G(f)\, df$$

The noise power spectral density at the output of the bandpass filter is given by

$$G_o(f) = G_{in}(f)\left|H_o(f)\right|^2$$

$\left|H_o(f)\right|$ is known. The input noise power spectral density must be obtained. The noise spectral density at the output of the multiplier may be calculated as

$$G_{om}(f) = F\{\phi_{om}(\tau)\}$$

(The subscript "om" refers to the output of the multiplier)

$$\phi_{om}(\tau) = \overline{e(t)\,e(t+\tau)}$$

where $e(t) = n'(t)\sqrt{2}\,C\,\cos(\omega_c t + \theta)$

$$\phi_{om}(\tau) = 2C^2\,\overline{n'(t)\,n'(t+\tau)\cos(\omega_c t + \theta)\cos(\omega_c(t+\tau) + \theta)}$$

Since n'(t) and $\cos\omega_c t$ are independent, we can write

$$\phi_{om}(\tau) = 2C^2\,\overline{n'(t)n'(t+\tau)} \times \overline{\cos(\omega_c t+\theta)\cos(\omega_c(t+\tau)+\theta)}$$

$$= 2C^2\left[\phi_{n'n'}(\tau) \times \phi_{cc}(\tau)\right]$$

By the convolution theorem

$$G_{om}(f) = F\{\phi_{om}(\tau)\} = 2C^2\int_{-\infty}^{\infty} g_{n'}(s)\,g_c(f-s)\,ds$$

where $g_{n'}(s)$ is the power spectral density of $n'(t)$ and $g_c(f-s)$ is the power spectral density of $\cos(\omega_c t+\theta)$. Since $g_c(f-s)$ is known,

it remains only to find the power spectral density of the noise input to the multiplier. This can be found from the power spectral density of the receiver input noise multiplied by $|H_i(f)|^2$.

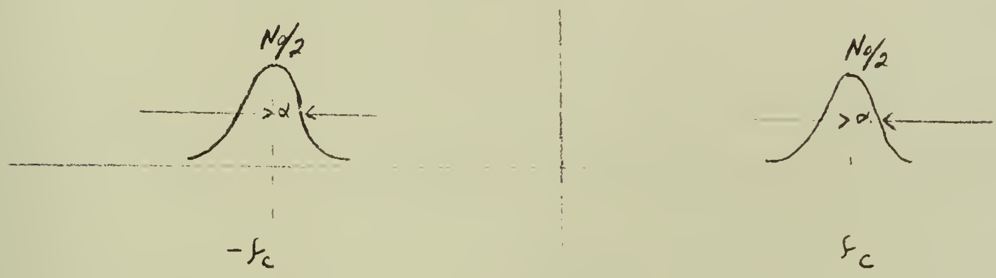
Therefore, the noise power at the output of the bandpass filter can be determined by the knowledge of the receiver input noise spectral density alone (assuming the constants of the receiver are known).

The power spectral density at the input to the multiplier may be calculated as

$$G_{iM}(f) = \frac{N_0}{2}|H_i(f)|^2 = \frac{N_0}{2\left(1+\left(\frac{f-f_c}{\alpha}\right)^2\right)} \qquad f \geq 0$$

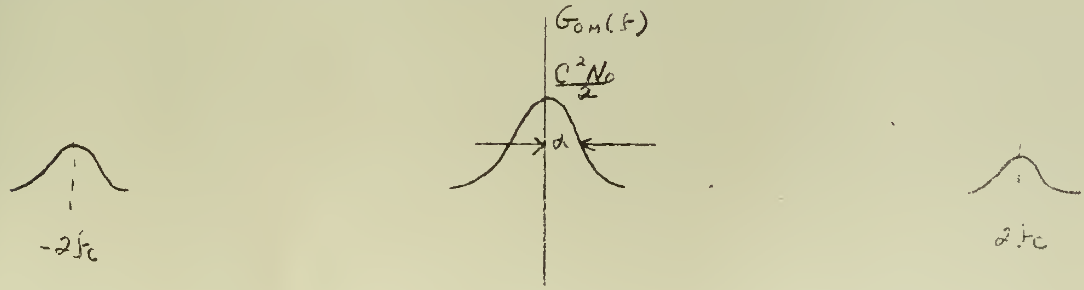
$$= \frac{N_0}{2\left(1+\left(\frac{f+f_c}{\alpha}\right)^2\right)} \qquad f \leq 0$$

A plot of $G_{iM}(f)$ is as follows



The convolution of this spectrum with the power spectrum of $\sqrt{2}\, C \cos(\omega_c t + \theta)$, i.e., $\frac{2C^2}{4}\left[\delta(f-f_c) + \delta(f+f_c)\right]$ will give the noise power spectrum at the output of the multiplier, $G_{oM}(f)$.

The convolution integral for these spectra can be carried out quite simply by graphical means to give



The low frequency portion of this spectrum may be written as

$$G_{OM}(f) = \frac{C^2 N_0}{2} \times \frac{1}{1+\left(\frac{f}{\alpha}\right)^2}$$

The spectrum at the output of the bandpass filter is

$$G_0(f) = G_{OM}(f) \left| H_0(f) \right|^2$$

$$= \frac{C^2 N_0}{2} \times \frac{1}{1+\left(\frac{f}{\alpha}\right)^2} \times \frac{1}{1+\left(\frac{f-f_m}{\beta}\right)^2} \qquad f \geq 0$$

$$= \frac{C^2 N_0}{2} \times \frac{1}{1+\left(\frac{f}{\alpha}\right)^2} \times \frac{1}{1+\left(\frac{f+f_m}{\beta}\right)^2} \qquad f \leq 0$$

Since $\beta << \alpha$, $G_{OM}(f)$ may be considered as constant at the value determined by f_m.

The output noise power is then

$$G_0(f) = \frac{C^2 N_0}{2\left(1+\left(\frac{f_m}{\alpha}\right)^2\right)} \left[\int_{-\infty}^{0} \frac{df}{1+\left(\frac{f+f_m}{\beta}\right)^2} + \int_{0}^{\infty} \frac{df}{1+\left(\frac{f-f_m}{\beta}\right)^2} \right]$$

Evaluation of the integrals may be accomplished by letting

$$u = \frac{f+f_m}{\beta}$$

$$du = \frac{df}{\beta} \qquad df = \beta\, du$$

when

$$f = 0 \qquad u = \frac{f_m}{\beta}$$

$$= -\infty \qquad u = -\infty$$

$$\int_{-\infty}^{\frac{f_m}{\beta}} \frac{\beta\, du}{1+u^2} = \beta \tan^{-1} u \Big]_{-\infty}^{\frac{f_m}{\beta}} = \beta \tan^{-1} \frac{f_m}{\beta} - \beta \tan^{-1} -\infty$$

$$\simeq \beta \pi \quad \text{for } \beta <<< f_m$$

The second integral can be evaluated in a similar manner to give the same result.

The output noise power is then

$$\frac{C^2 N_0}{2\left(1+\left(\frac{f_m}{\alpha}\right)^2\right)}\left[\beta\pi+\beta\pi\right] = \frac{\beta\pi N_0 C^2}{1+\left(\frac{f_m}{\alpha}\right)^2}$$

The ratio of output signal power to output noise power is

$$\frac{\frac{1}{2}\left(\frac{mAC}{2}\right)^2(\cos\theta)^2}{\frac{\beta\pi N_0 C^2}{1+\left(\frac{f_m}{\alpha}\right)^2}} = \frac{m^2A^2}{8}\times\frac{\left(1+\left(\frac{f_m}{\alpha}\right)^2\right)}{\beta\pi N_0}\cos^2\theta$$

$$= \frac{1}{8}\frac{m^2A^2}{\beta\pi N_0}\left(1+\left(\frac{f_m}{\alpha}\right)^2\right)\cos^2\theta$$

For $f_m \ll \alpha$, the receiver output $\frac{S}{N}$ is

$$\left[\frac{m^2\cos^2\theta}{8\pi}\right]\left[\frac{A^2}{\beta N_0}\right]$$

The output $\frac{S}{N}$ ratio is dependent upon the post detection bandwidth and not predetection bandwidth for all values of input signal to noise ratios.

BIBLIOGRAPHY

1. Cubic Corporation, Study Phase Engineering Report, Project P-785, AF08(606)-785, June, 1955.

2. Cubic Corporation, Final Engineering Report, DME, Feb., 1956.

3. R. M. Fano, Signal to Noise Ratios in Correlation Detectors, MIT Research Lab of Electronics, Tech. Rep. 186, Feb., 1951.

4. Jaffe and Rechtin, Design and Performance of Phase-Lock Circuits Capable of New-Optimum Performance over a Wide Range of Input Signal and Noise Levels, I.R.E. Transactions on Information Theory, Vol. IT-1, pp. 66-76, March, 1955.

5. R. C. Patrick, A Wide-Band 550 Megacycle Amplifier, Proc. I.R.E., Vol. 35, pp. 1371-1374, Nov., 1947.

6. D. Richman, Color-Carrier Reference Phase Synchronization Accuracy in NTSC Color Television, Proc. I.R.E., Vol. 42, pp. 106-133, Jan., 1954.

7. D. Richman, The D.C. Quadricorrelator: A Two-Mode Synchronization System, Proc. I.R.E., Vol. 24, pp. 288-299, Jan., 1954.

8. J. B. Thomas, On the Statistical Design of Demodulation Systems for Signals in Additive Noise, Stanford University, Electronics Research Laboratory, Tech. Rep. No. 88, Aug., 1955.

9. P. G. Tucker, The History of the Homodyne and Synchrodyne, Journal of the British Inst. of Radio Engineers, pp. 143-154, Apr., 1954.

10. P. M. Woodward and I. L. Davies, Information Theory and Inverse Probability in Telecommunications, Proc. IRE, pp. 37-44, March 1952.

11. P. M. Woodward, Probability and Information Theory, McGraw-Hill Book Co., 1953.

Thesis
S448 Sheridan
35726
Application of correlation detection to distance measuring equipment.